Do You Do This?

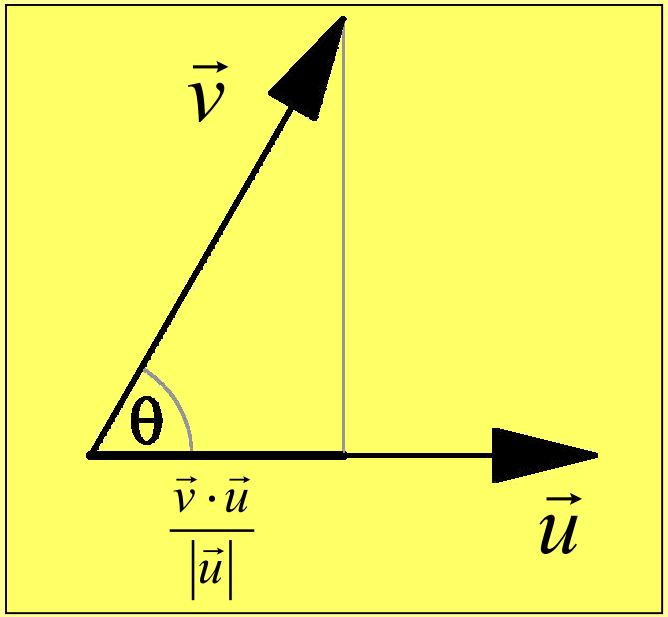
$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

$$\left|\vec{u}\right| = \sqrt{u_1^{\,2} + u_2^{\,2} + u_3^{\,2}}$$

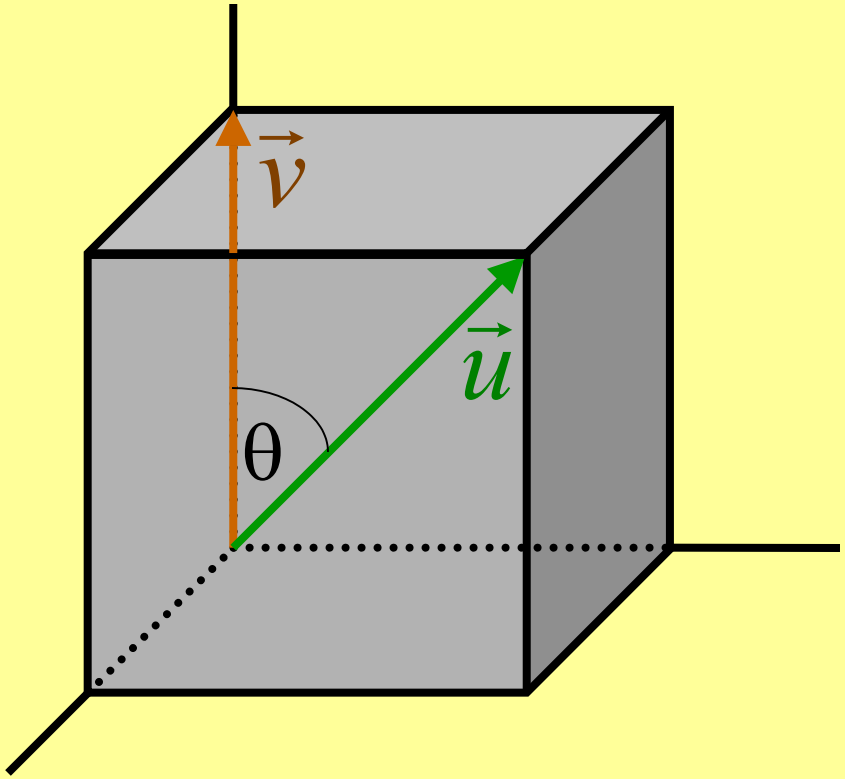
Or This?

$$\vec{u}\cdot\vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

$$\left|\vec{u}\right| = \sqrt{\vec{u}\cdot\vec{u}}$$



The Cube



$$\vec{u} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{v} = \hat{k}$$

$$\cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{\left|\vec{u}\right|\left|\vec{v}\right|} = \frac{1}{\sqrt{3}}$$

Find the angle between a diagonal of a cube and an edge

The Cube

- **Emphasizes that vectors are arrows**
- **Combines geometry and algebra**
- **Uses multiple representations**

geometry: $\vec{u} \cdot \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right| \cos\theta$

algebra: $\vec{u} \cdot \vec{v} = \left(u_1 \hat{\imath} + u_2 \hat{\jmath} + u_3 \hat{k}\right) \cdot \left(v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}\right)$

memory: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Multiple Representations

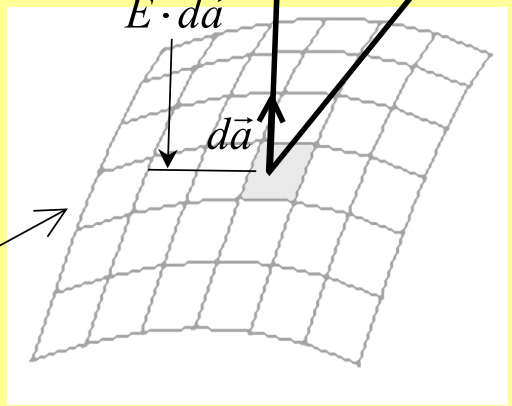
Flux is the total amount of electric field through a given area.

Perpendicular Component

Normal Vector

$$\Phi = \int \vec{E} \cdot d\vec{a}$$

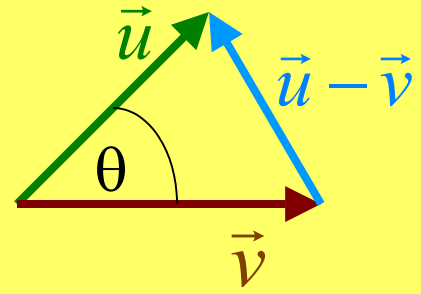
$\sum$ over all rectangles



Start with Algebra?

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

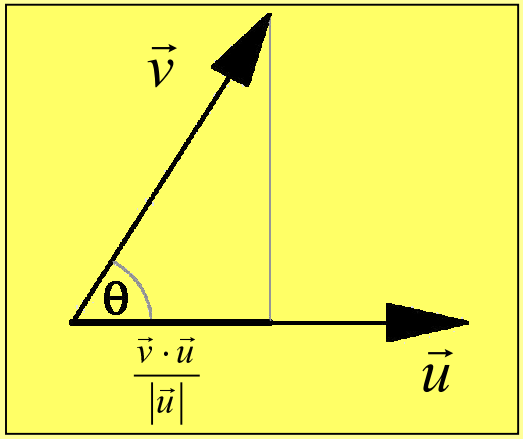
$$\Rightarrow (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} + \vec{v}\cdot\vec{v} - 2\vec{u}\cdot\vec{v}$$

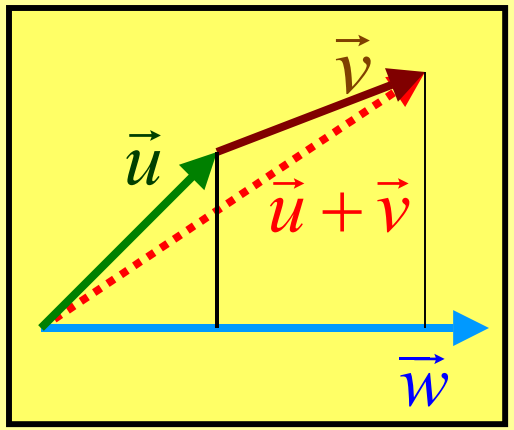


(Law of Cosines)

$$\left|\vec{u}-\vec{v}\right|^2 = \left|\vec{u}\right|^2 + \left|\vec{v}\right|^2 - 2\left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

Start with Geometry!



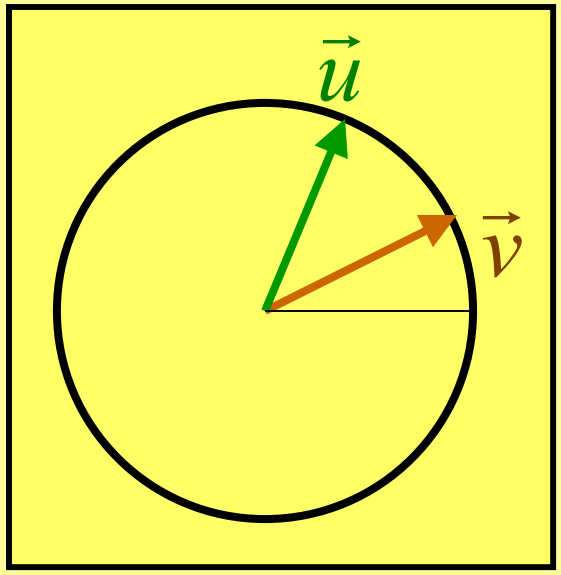


$$\Rightarrow (\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w}+\vec{v}\cdot\vec{w}$$

$$\Rightarrow \left(u_1\hat{\imath}+u_2\hat{\jmath}\right)\cdot\left(v_1\hat{\imath}+v_2\hat{\jmath}\right) = u_1v_1+u_2v_2$$

(get Law of Cosines for free!)

Use both!



$$\vec{u} = \cos(\alpha)\,\hat{\imath} + \sin(\alpha)\,\hat{\jmath}$$

$$\vec{v} = \cos(\beta)\,\hat{\imath} + \sin(\beta)\,\hat{\jmath}$$

$$\vec{u} \cdot \vec{v} = \cos(\alpha - \beta)$$

(get addition formulas for free!)