Bridging the G A P between Mathematics and Physics

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http://www.physics.oregonstate.edu/bridge

Support

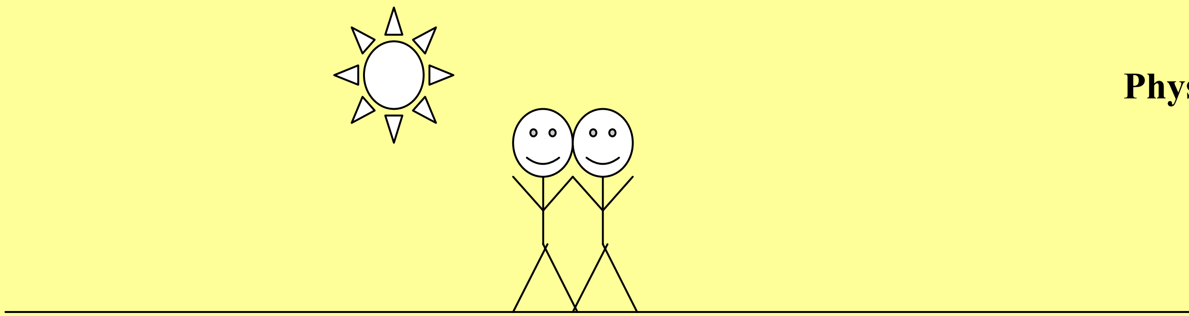
- **Mathematical Association of America**
  - **Professional Enhancement Program**
- **Oregon State University**
  - **Department of Mathematics**
  - **Department of Physics**
- **National Science Foundation**
  - **DUE-9653250**
  - **DUE-0088901**
  - **DUE-0231032**
  - **DUE-0231194**

Support

- **Grinnell College**
  - **Noyce Visiting Professorship**
- **Mount Holyoke College**
  - **Hutchcroft Fund**
- **Oregon Collaborative for Excellence in the Preparation of Teachers**

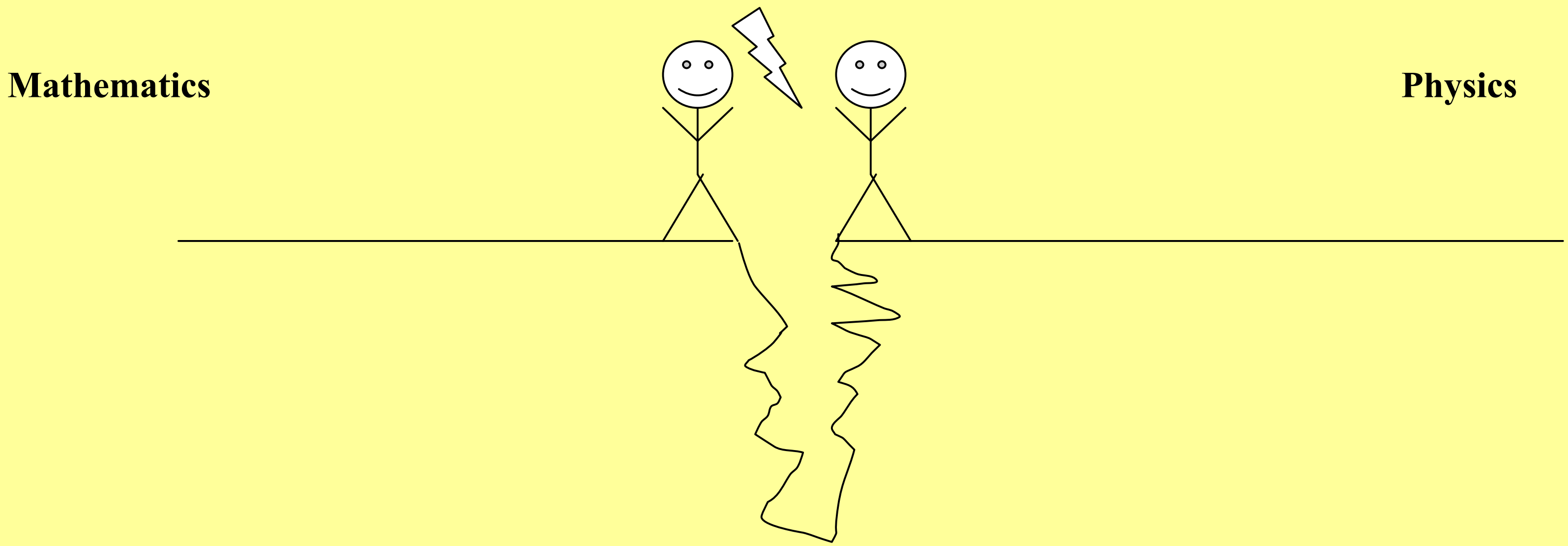
Mathematics Physics

Mathematics



Physics

The Grand Canyon



What Are Functions?

$$T(x,y) = k(x^2 + y^2)$$

$$T(r,\theta) = ?$$

What Are Functions?

Suppose the temperature on a rectangular slab of metal is given by:

$$T(x,y) = k(x^2 + y^2)$$

Physics: $T(r,\theta) = k\,r^2$

Math: $T(r,\theta) = k(r^2 + \theta^2)$

What Are Functions?

Math:

$$T = f(x, y) = k(x^2 + y^2)$$

$$T = g(r, \theta) = k\,r^2$$

Physics:

$$T = T(x, y) = k(x^2 + y^2)$$

$$T = T(r, \theta) = k\,r^2$$

Mathematics $\neq$ Physics

- **Physics is about things.**
- **Physicicists can't change the problem.**

Physics is about things

- What sort of a beast is it?
  - Scalar fields, units, coordinates, time.
- Physics is independent of coordinates.
  - Vectors as arrows, geometry of dot and cross.
- Graphs are about relationships of physical things.
  - Fundamental physics is 3 dimensional.
  - 3-d graphs of functions of 2-variables are misleading.
  - Hills are not a good example of functions of 2 variables.
  - Use of color.
- Fundamental physics is highly symmetric.
  - Spheres and cylinders vs. parabolas.
  - Interesting physics problems can involve trivial math.
  - Adapted basis vectors, such as $\hat{r}, \hat{\theta}, \hat{\phi}$.

Physicists can't change the problem.

- Physics involves creative synthesis of multiple ideas.
- Physics problems not nec. well-defined math problems.
  - No preferred coordinates or independent variables.
  - No parameterization.
  - Unknowns don't have names.
  - Getting to a well-defined math problem is part of the problem.
  - If you can't add units, it's a poor physics problem.
- Physics problems don't fit templates.
  - Template problem-solving vs. skills.
  - A few key ideas are remembered best later.
- Physics involves interplay of multiple representations.
  - Dot product.

Differentials

- **Substitution**

$$\int 2x \sin x \, dx$$

$$u = x^2$$

$$du = 2x \, dx$$

- **Chain Rule**

$$x = \cos\theta$$

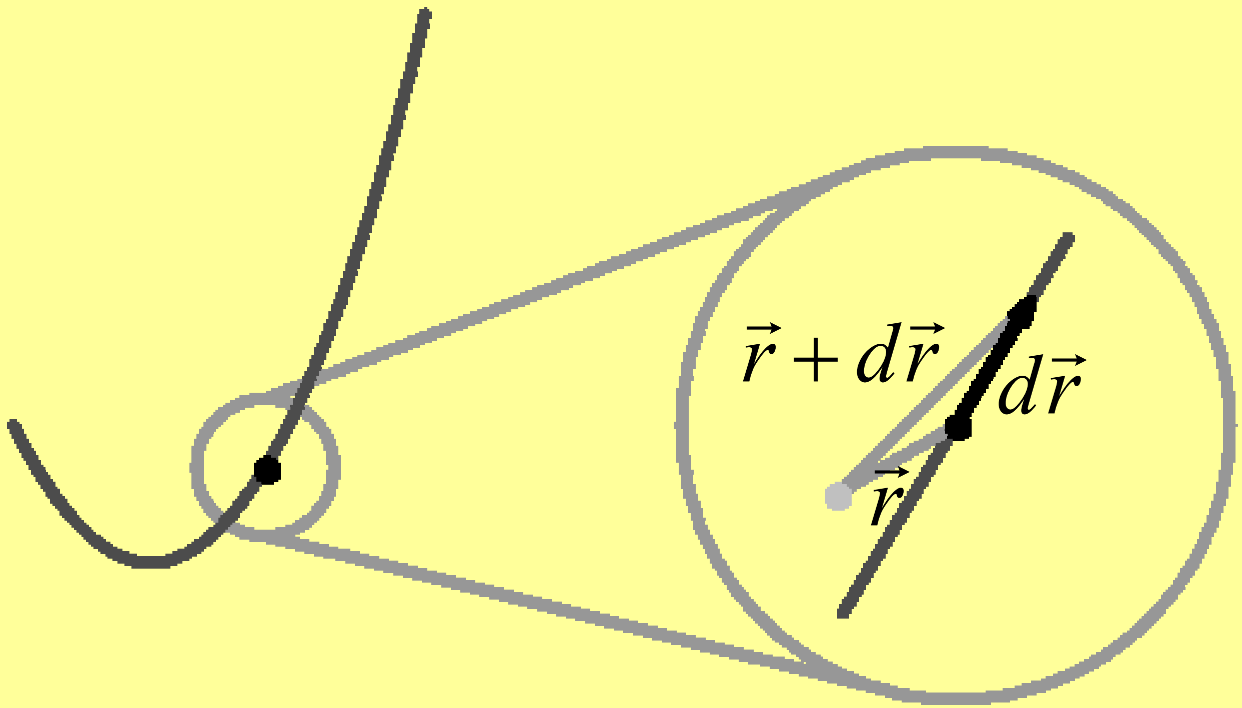
$$dx = -\sin\theta \, d\theta$$

$$u = x^2$$

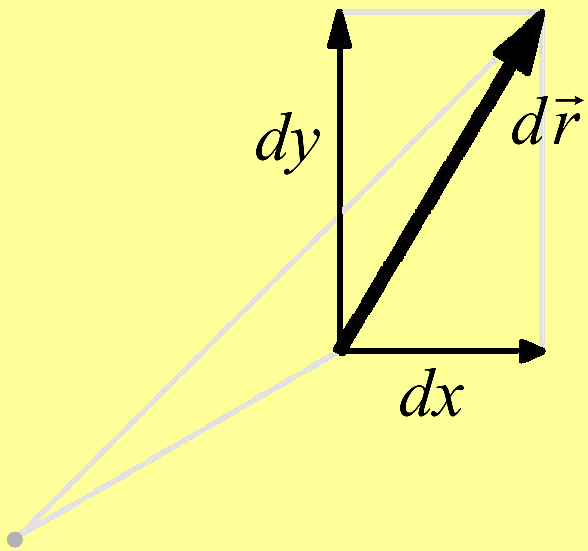
$$du = 2x \, dx$$

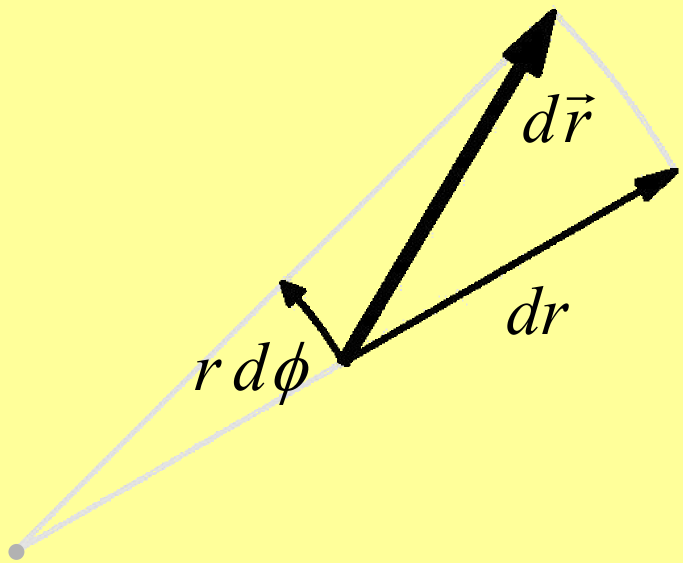
$$= 2\cos\theta(-\sin\theta \, d\theta)$$

Vector Differentials



Vector Differentials

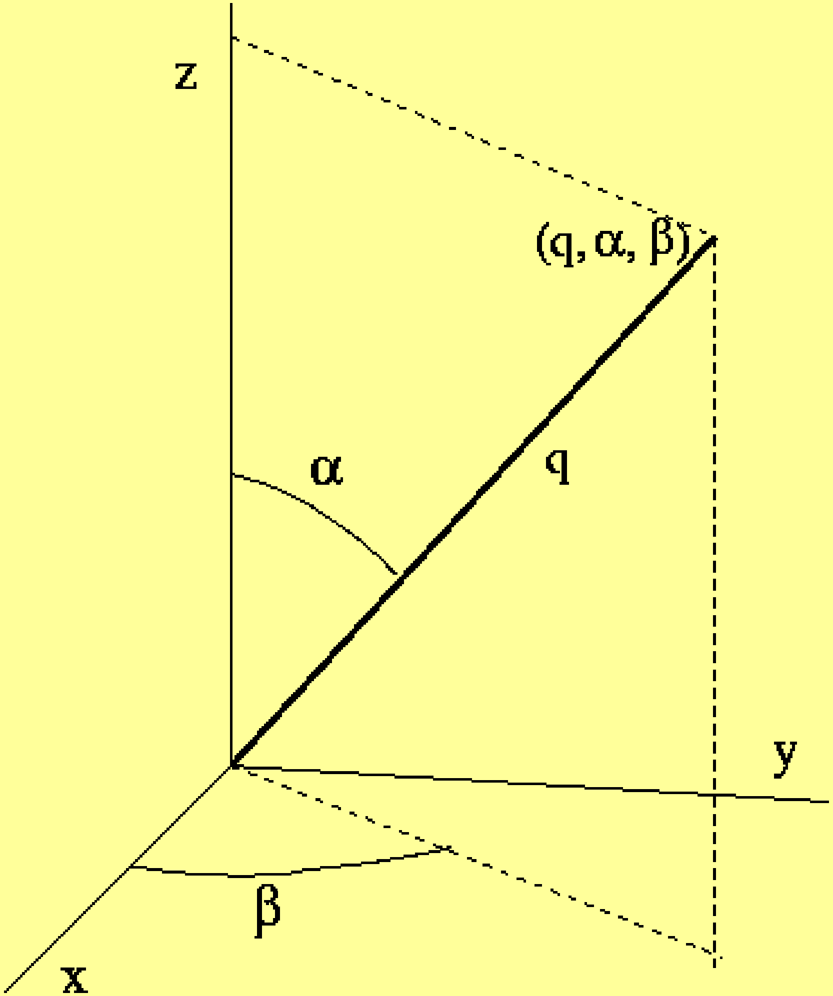




$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$d\vec{r} = dr\,\hat{r} + r\,d\phi\,\hat{\phi}$$

Spherical Coordinates



q: Radius

α: Zenith (colatitude)

β: Azimuth (longitude)

Math: (ρ, φ, θ)

Physics: (r, θ, φ)

Vector Differentials

- **Line Integrals**

$$\int \vec{F} \cdot d\vec{r}$$

$$\int f\, ds \qquad ds = \left| d\vec{r} \right|$$

- **Surface Integrals**

$$\int\int \vec{F} \cdot d\vec{S} \qquad d\vec{S} = d\vec{r}_1 \times d\vec{r}_2$$

$$\int\int f\, dS \qquad dS = \left| d\vec{r}_1 \times d\vec{r}_2 \right|$$

- **Volume Integrals**

$$dV = (d\vec{r}_1 \times d\vec{r}_2) \cdot d\vec{r}_3$$

- **Gradient**

$$df = \vec{\nabla} f \cdot d\vec{r}$$

In a Nutshell

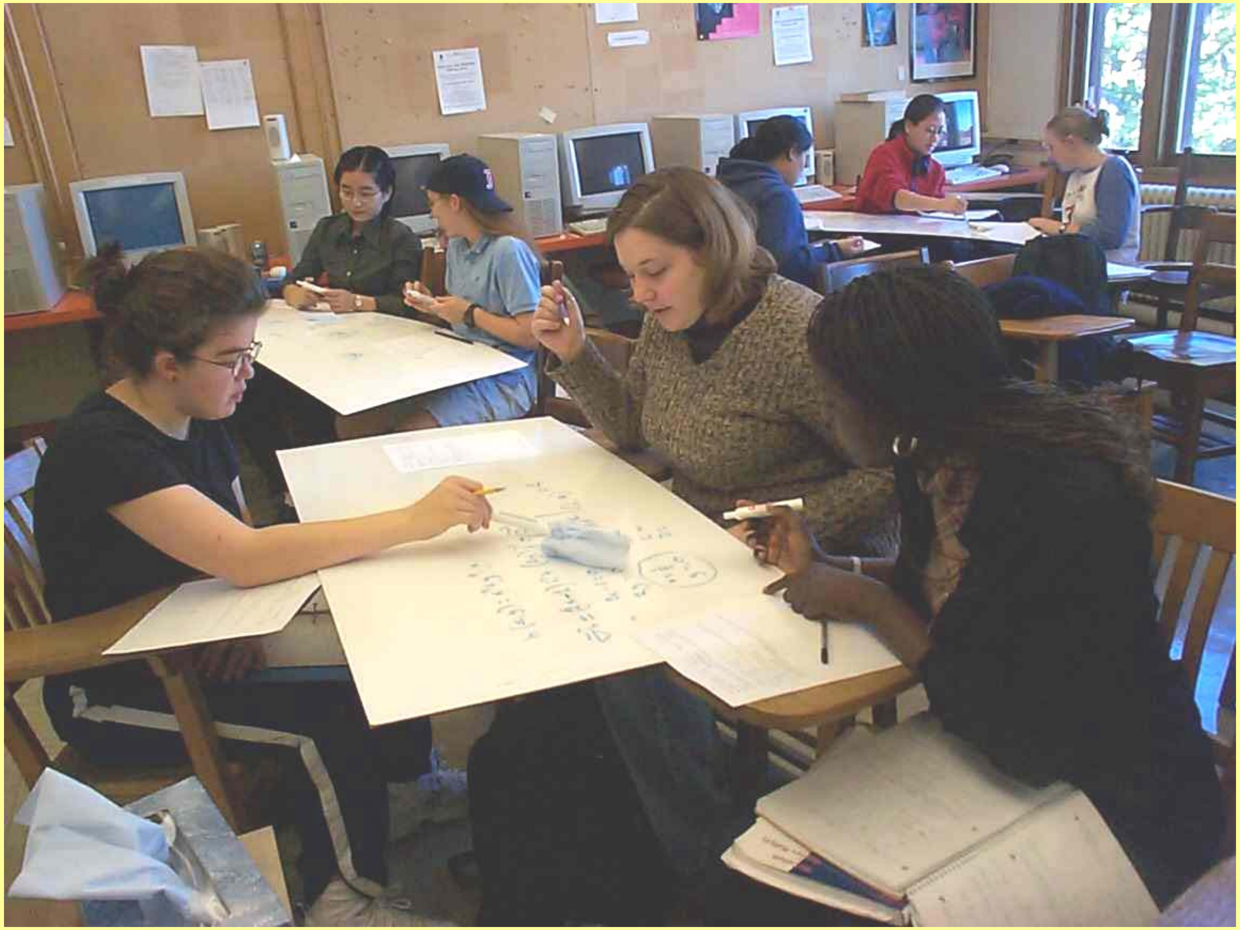
***Geometric visualization
is the key to bridging the gap
between mathematics
and physics.***

Context Rich Problems

Just as you turn onto the main avenue from a side street with a stop sign, a city bus going 30-mph passes you in the adjacent lane. You want to get ahead of the bus before the next stoplight which is two blocks away. Each block is 200-ft long and the side streets are 25-ft wide, while the main avenue is 60-ft wide. If you increase your speed at a rate of 5-mph each second, will you make it? (No Picture)

Patricia Heller and Kenneth Heller, University of Minnesota

Interactive Classroom



Lecture vs. Activities

- **The Instructor:**
  - Paints big picture.
  - Inspires.
  - Covers lots fast.
  - Models speaking.
  - Models problem-solving.
  - Controls questions.
  - Makes connections.
- **The Students:**
  - Focus on subtleties.
  - Experience delight.
  - Slow, but in depth.
  - Practice speaking.
  - Practice problem-solving.
  - Control questions.
  - Make connections.

Socratic vs. Groups

How does it feel to teach in these ways?

$$\int_{class} d\ knowledge \quad \text{vs.} \quad \int_{class} d\ questions$$

Everyone knows everything vs. No one knows anything

Group Activities

- **Task Master**

  Keeps group on track:

  "What you had for lunch doesn't seem relevant."

- **Cynic**

  Questions everything:

  "Why?" "Why?" "Why?"

- **Recorder**
- **Reporter**

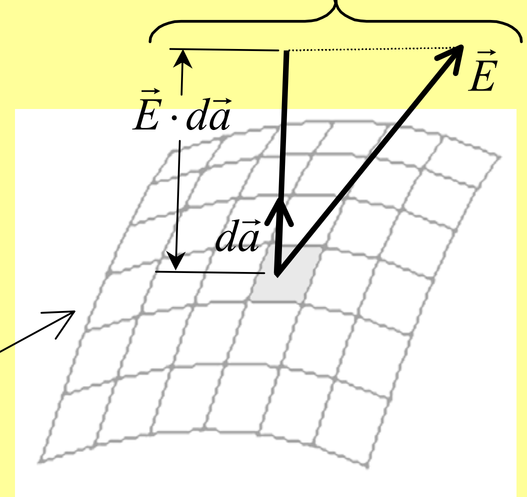
Multiple Representations

Flux is the total amount of electric field through a given area.

Perpendicular Component

Normal Vector

$$\Phi = \int \vec{E} \cdot d\vec{a}$$



$\sum$ over all rectangles

Do You Do This?

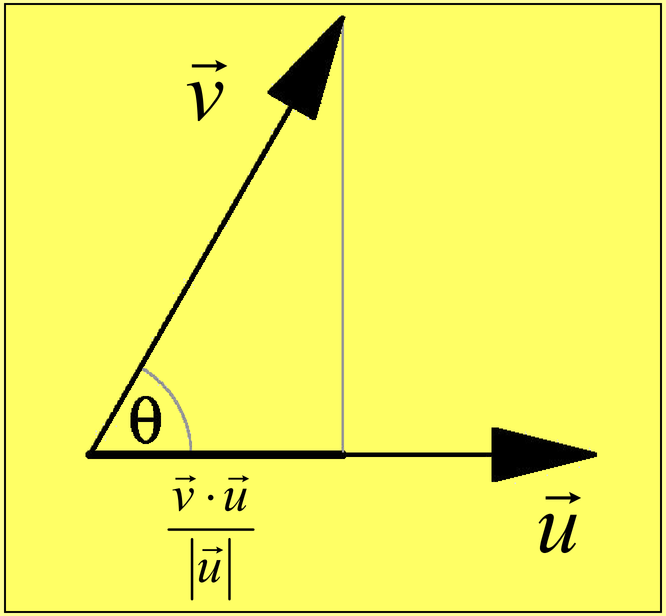
Or This?

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

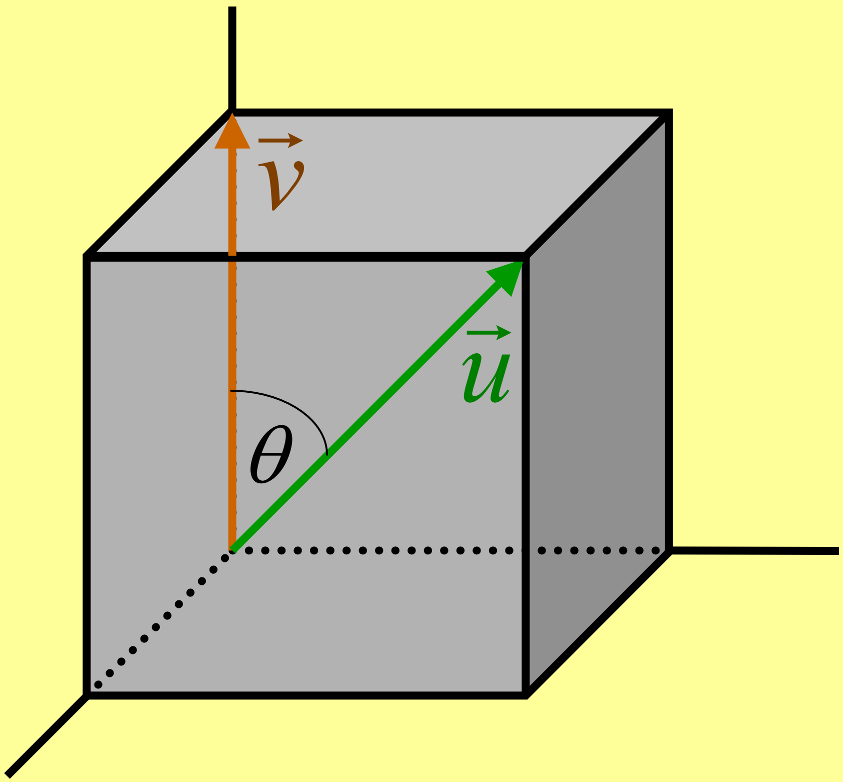
$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$|\vec{u}| = \sqrt{u_1{}^2 + u_2{}^2 + u_3{}^2}$$

$$|\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}}$$



The Cube



$$\vec{u} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{v} = \hat{k}$$

$$\cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{\left|\vec{u}\right|\left|\vec{v}\right|} = \frac{1}{\sqrt{3}}$$

Find the angle between a diagonal of a cube and an edge

The Cube

- **Emphasizes that vectors are arrows**
- **Combines geometry and algebra**
- **Uses multiple representations**

geometry: $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

algebra: $\vec{u} \cdot \vec{v} = \left(u_1 \hat{\imath} + u_2 \hat{\jmath} + u_3 \hat{k}\right) \cdot \left(v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}\right)$

memory: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$