# BRIDGING THE VECTOR CALCULUS GAP


*Tevian Dray & Corinne Manogue*

**Oregon State University**


## The Bridge Project

Differentials (Use what you know!)
Exploiting multiple representations
Symmetry (adapted bases, coordinates)
Geometry (vectors, div, grad, curl)
Computer visualization (Maple, Excel)
Active engagement (groups, flashcards)

## Support


- Oregon State University
  - Department of Mathematics
  - Department of Physics
- Grinnell College
  - Noyce Visiting Professorship
- Mount Holyoke College
  - Hutchcroft Fund
- National Science Foundation
  - DUE-9653250
  - DUE-0088901
- Oregon Collaborative for Excellence in the Preparation of Teachers


## A Radical View of Calculus

- The central idea of calculus is not the limit.
- The central idea of derivatives is not slope.
- The central idea of integrals is not area.
- The central idea of curves and surfaces is not parameterization.
- The central representation of a function is not its graph.

## Do You Do This?

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

$$|\vec{u}| = \sqrt{u_1^{\,2} + u_2^{\,2} + u_3^{\,2}}$$

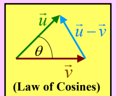
## Or This?

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta \qquad |\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}}$$

## Start with Algebra?

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

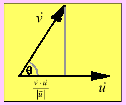
$$\Rightarrow (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} + \vec{v}\cdot\vec{v} - 2\vec{u}\cdot\vec{v}$$

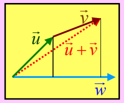


(Law of Cosines)

$$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos\theta$$

## Start with Geometry!



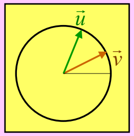




$$\Rightarrow (\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w}$$

$$\Rightarrow (u_1\hat{\imath} + u_2\hat{\jmath})\cdot(v_1\hat{\imath} + v_2\hat{\jmath}) = u_1v_1 + u_2v_2$$

(get Law of Cosines for free!)

## Use both!




$$\vec{u} = \cos(\alpha)\hat{\imath} + \sin(\alpha)\hat{\jmath}$$

$$\vec{v} = \cos(\beta)\hat{\imath} + \sin(\beta)\hat{\jmath}$$

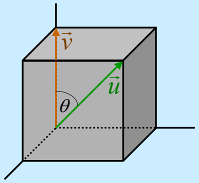
$$\vec{u}\cdot\vec{v} = \cos(\alpha-\beta)$$

(get addition formulas for free!)

## Group Activities

- Task Master
  **Keeps group on track:**
  *"What you had for lunch doesn't seem relevant."*
- Cynic
  **Questions everything:**
  *"Why?" "Why?" "Why?"*
- Recorder
- Reporter

## Sample Group Activity




$$\vec{u} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{v} = \hat{k}$$

$$\cos(\theta) = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{1}{\sqrt{3}}$$

Find the angle between a diagonal of a cube and an edge

## Group Activity

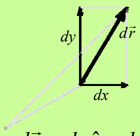
Emphasizes that vectors are arrows
Combines geometry and algebra
Uses multiple representations

geometry: $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

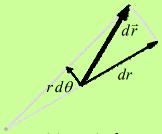
algebra: $\vec{u}\cdot\vec{v} = (u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k})\cdot(v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k})$

memory: $\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$

## Vector Differentials




$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$




$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

**Line Integrals**

$$\int \vec{F}\cdot d\vec{r}$$

$$\int f\,ds \qquad ds = |d\vec{r}|$$

**Volume Integrals**

$$dV = (d\vec{r}_1 \times d\vec{r}_2)\cdot d\vec{r}_3$$

**Surface Integrals**

$$\iint \vec{F}\cdot d\vec{S} \qquad d\vec{S} = d\vec{r}_1 \times d\vec{r}_2$$

$$\iint f\,dS \qquad dS = |d\vec{r}_1 \times d\vec{r}_2|$$

**Gradient**

$$df = \vec{\nabla} f\cdot d\vec{r}$$