Bridging the Gap

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http://www.physics.oregonstate.edu/bridge
http://www.physics.oregonstate.edu/paradigms

Support

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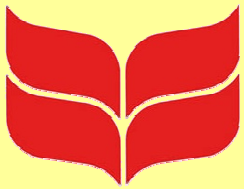
- Oregon State University
- Grinnell College
  - Noyce Visiting Professorship
- Mount Holyoke College
  - Hutchcroft Fund



It is not true that students fail to learn mathematics in their mathematics courses.

Things Students DO Learn

- Derivatives as slopes (and rates of change).
- How to find derivatives.
- Integrals as areas.
- Substitutions in integrals (including limits).
- Taylor series coefficients.
- Algebraic definitions of dot & cross.
- Gradients—uphill, perpendicular to level curves.
- To be careful—definitions, singularities, limits.

Things Students Do NOT Learn

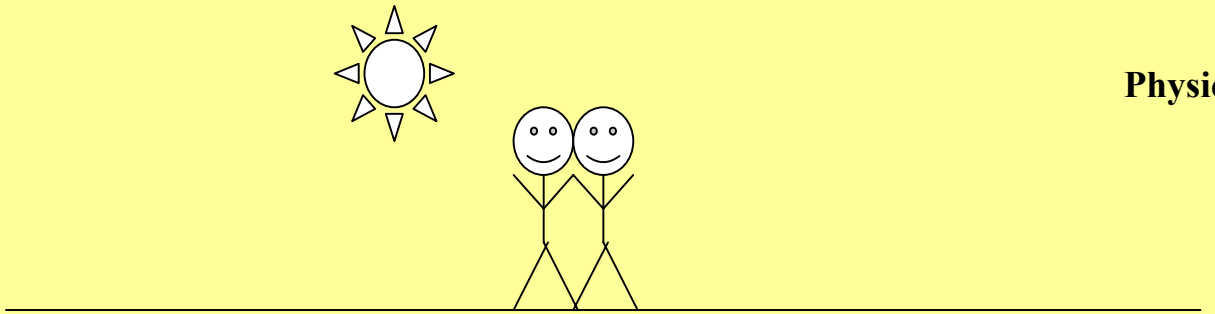
- Algebra & Trig (but you know this!)
- Integrals as chopping something up.
- Power series as approximations.
- Geometric definitions of dot and cross,
  - esp. dot of perpendicular vectors.
  - distinguish between dot and cross.
- Advanced vector calculus topics.
- To be confident.

Why is the transition from lower-division mathematics to upper-division physics so difficult for students?

Mathematics

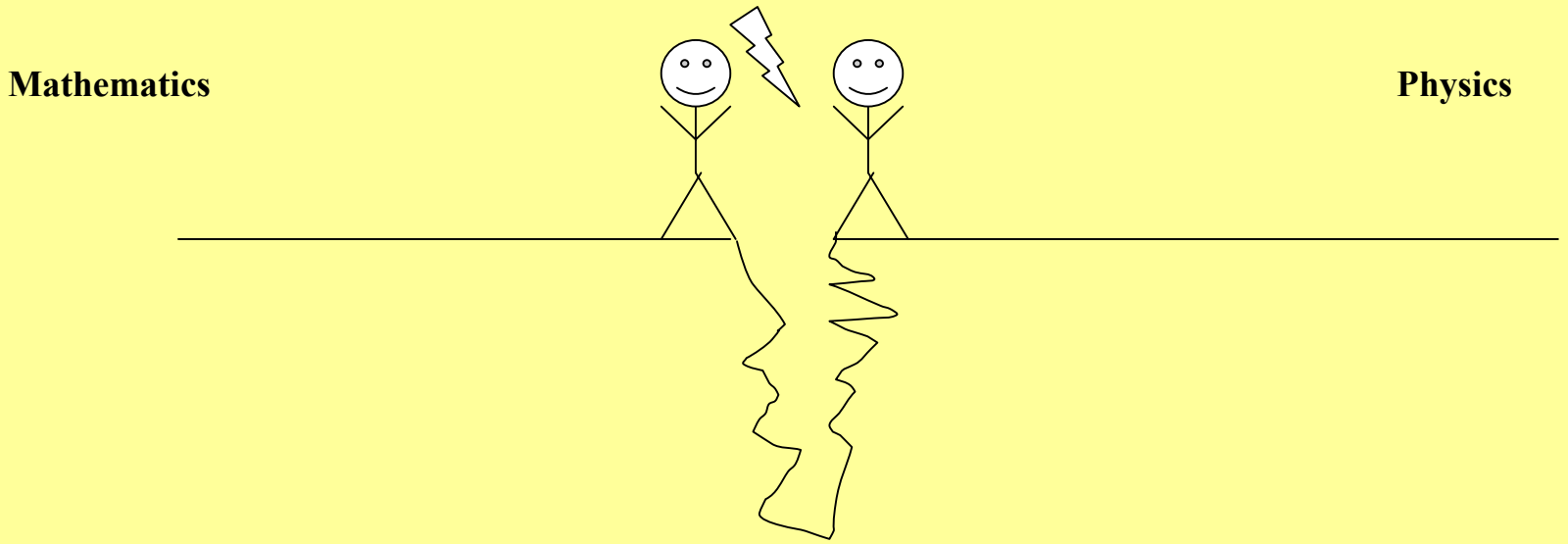
Physics

Mathematics



Physics

The Grand Canyon

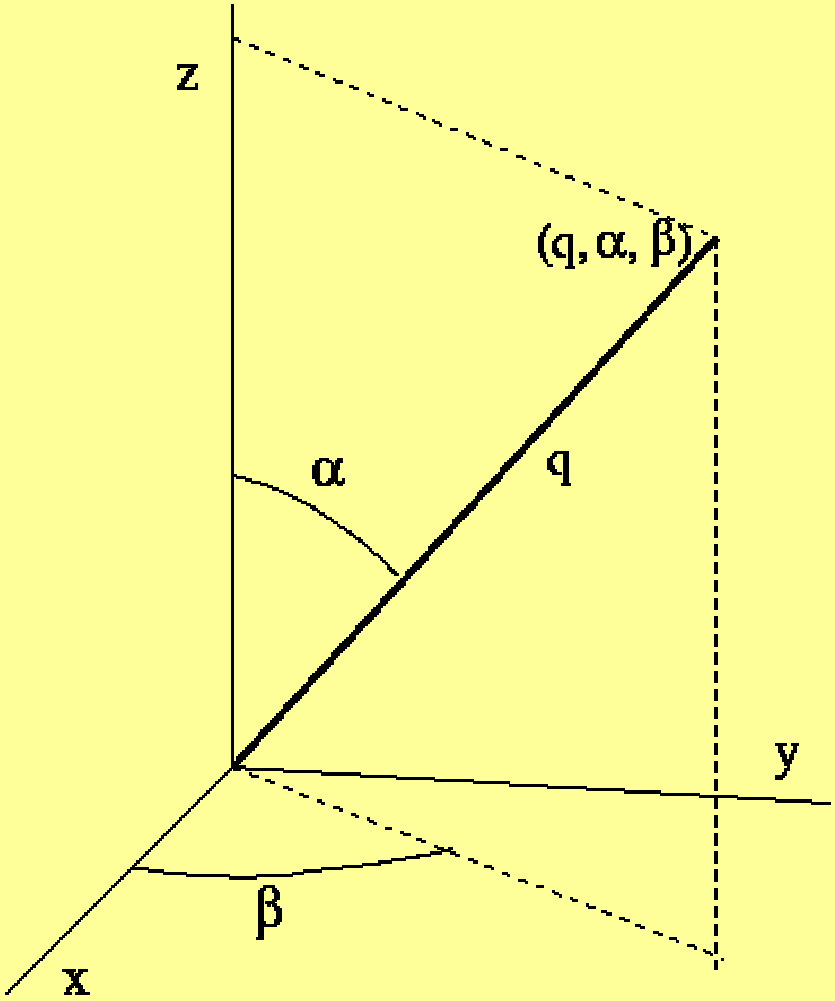


Outline

- Differences in notation.
- Physics is about things.
- Multiple representations.
- The content that no one owns.
- Active engagement.

Differences in Notation

Spherical Coordinates



q: Radius

α: Zenith (colatitude)

β: Azimuth (longitude)

Math: (ρ, ϕ, θ)

Physics: (r, θ, ϕ)

Which Way is…?

$\hat{r}$

Which Way is…?

$$\hat{\theta}$$

Physics Is About Things

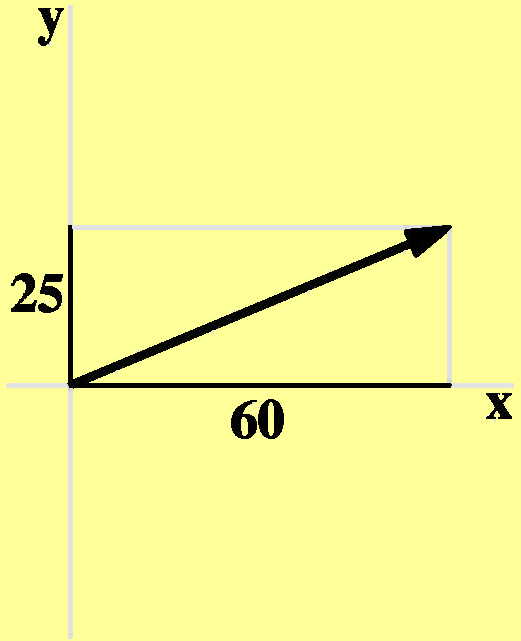
- What sort of a beast is it?
- Physics is independent of coordinates.
- Graphs are about the relationships of physical things to other physical things.
- Fundamental physics is highly symmetric.

What Sort of a Beast is It?

Mathematics:

- Triples of numbers

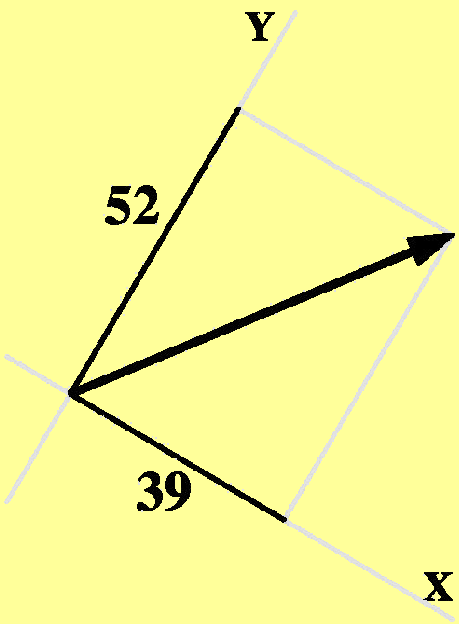
$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$



Physics:

- Arrows in space

$$\vec{v} = v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}$$



A Question for the Audience

$$T(x,y)=k(x^2+y^2)$$

$$T(r,\theta)=?$$

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$$T(x,y)=k(x^2+y^2)$$

$$T(r,\theta)=?$$

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How Do Things Change?

- Coulomb's Law:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

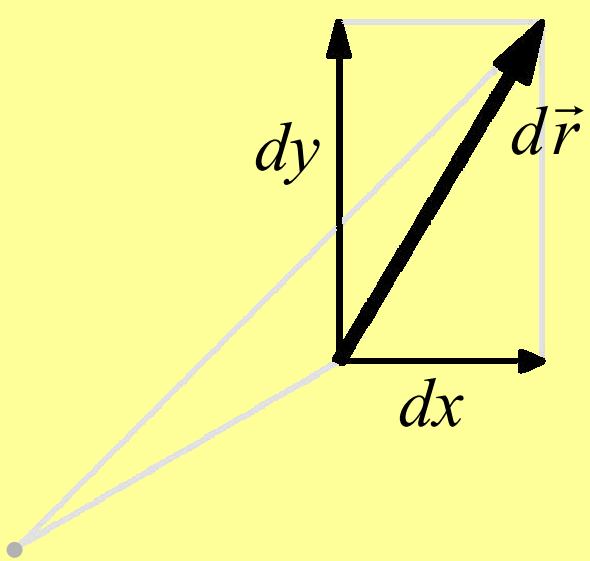
Central Forces

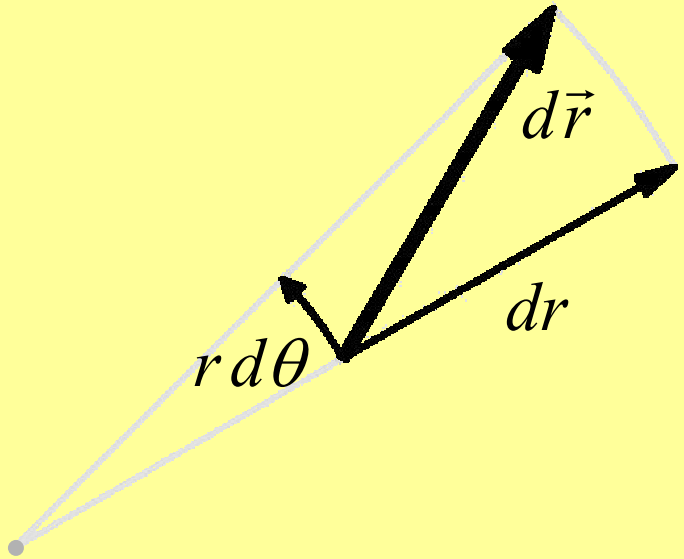
The Ring

i.e. Graphs Are About the Relationships of Physical Quantities to Other Physical Quantities.

Fundamental Physics is Highly Symmetric

Integrals

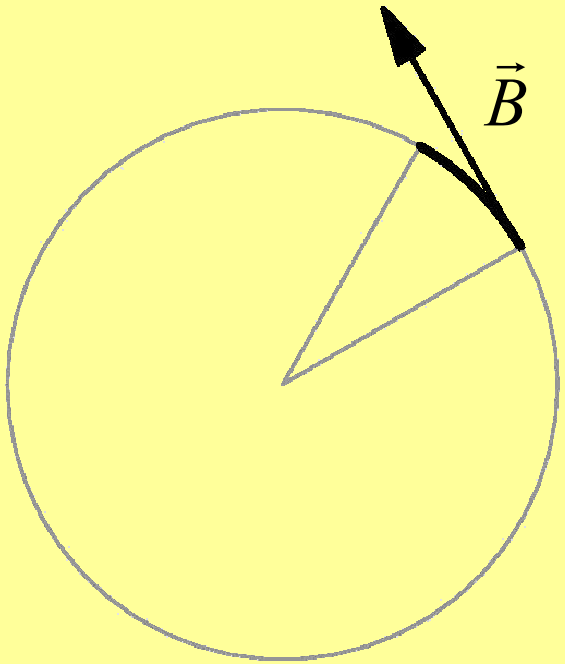




$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

Symmetry



$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r}\hat{\theta}$$

$$d\vec{r} = r\,d\theta\,\hat{\theta}$$

$$\int \vec{B}\cdot d\vec{r} = \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\theta = \mu_0 I$$

Ignoring Symmetry

$$\vec{B} = \frac{\mu_0 I}{2\pi} \frac{-y\,\hat{\imath} + x\,\hat{\jmath}}{x^2 + y^2}$$

$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned} \quad \Rightarrow \quad \begin{aligned} dx &= -r\sin\theta\, d\theta \\ dy &= r\cos\theta\, d\theta \end{aligned}$$

$$\int \vec{B} \cdot d\vec{r} = \int_0^{2\pi} \frac{\mu_0 I}{2\pi} d\theta = \mu_0 I$$

Multiple Representations

Do You Do This?

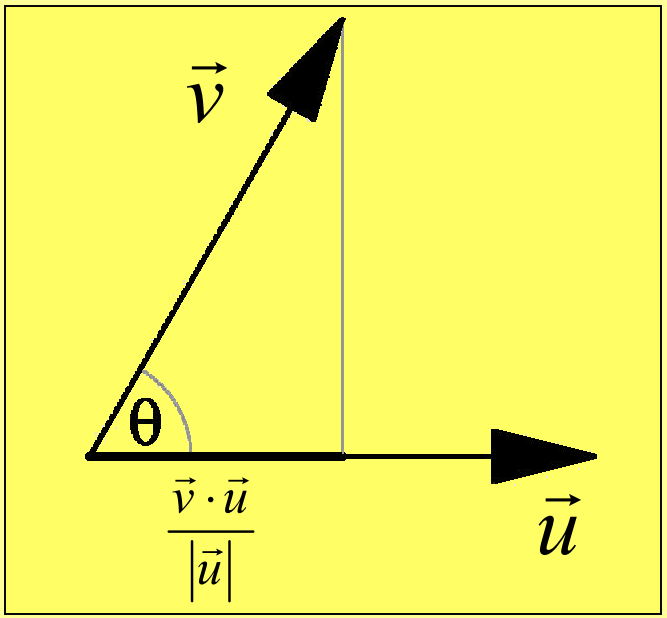
$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

$$\left|\vec{u}\right| = \sqrt{u_1{}^2 + u_2{}^2 + u_3{}^2}$$

Or This?

$$\vec{u}\cdot\vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

$$\left|\vec{u}\right| = \sqrt{\vec{u}\cdot\vec{u}}$$



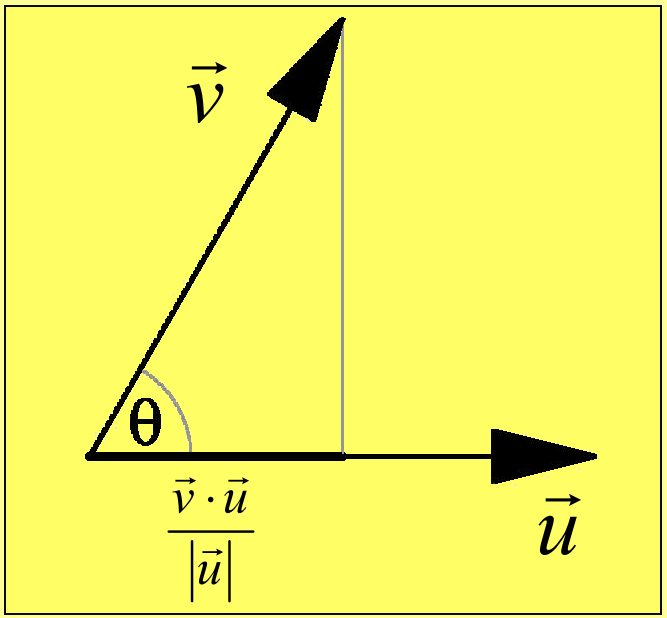
Algebra or Geometry?

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

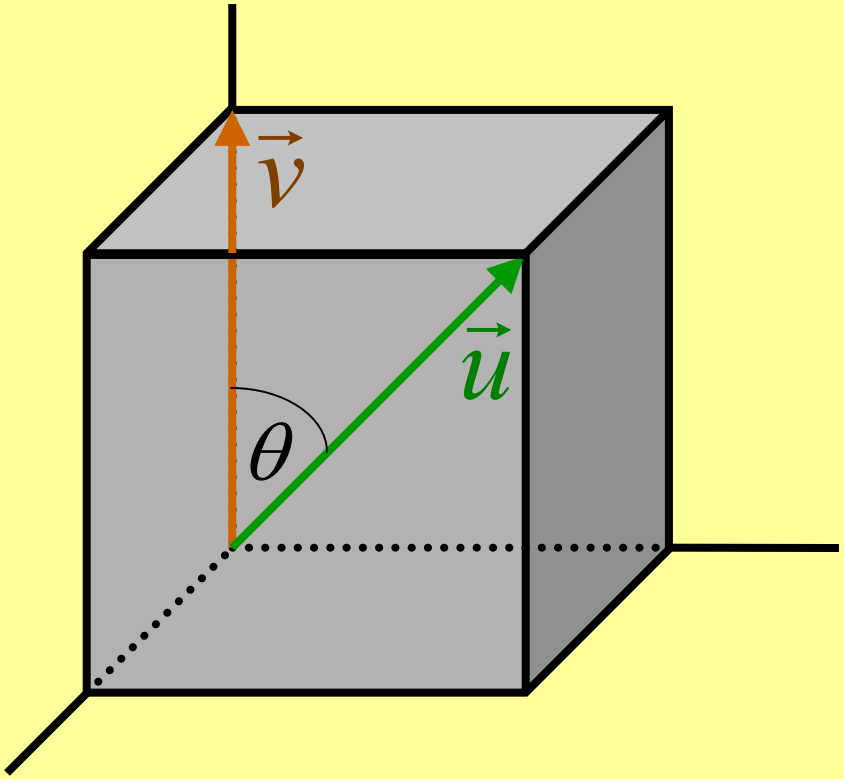
$$\vec{u}\cdot\vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

$$\left|\vec{u}\right| = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$$

$$\left|\vec{u}\right| = \sqrt{\vec{u}\cdot\vec{u}}$$



Sample Group Activity



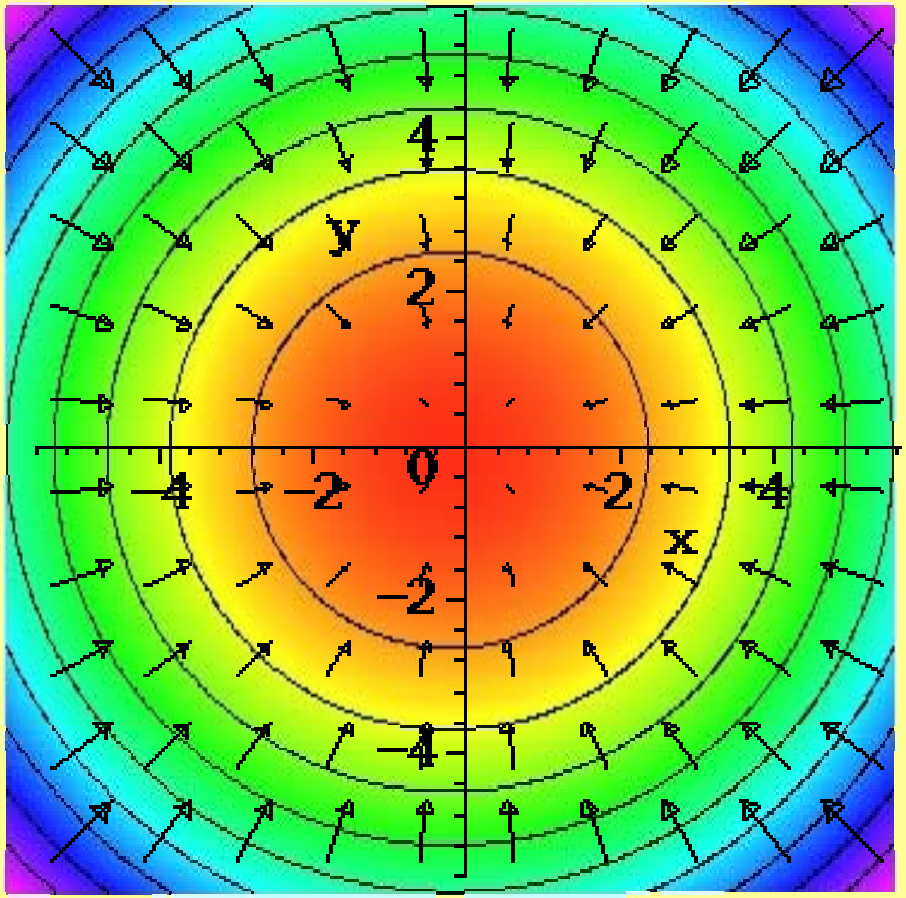
$$\vec{u} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{v} = \hat{k}$$

$$\cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{\left|\vec{u}\right|\left|\vec{v}\right|} = \frac{1}{\sqrt{3}}$$

Find the angle between a diagonal of a cube and an edge

Where Is the Top of the Mountain?



$$\vec{\nabla} f = 0$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = 0$$

Multiple Representations

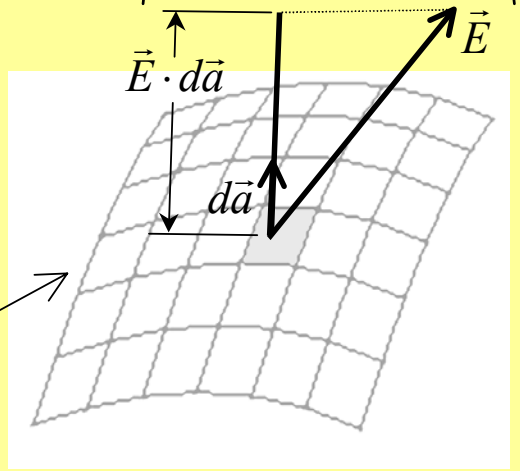
Flux is the total amount of electric field through a given area.

Perpendicular Component

Normal Vector

$$\Phi = \int \vec{E} \cdot d\vec{a}$$

$\sum$ over all rectangles

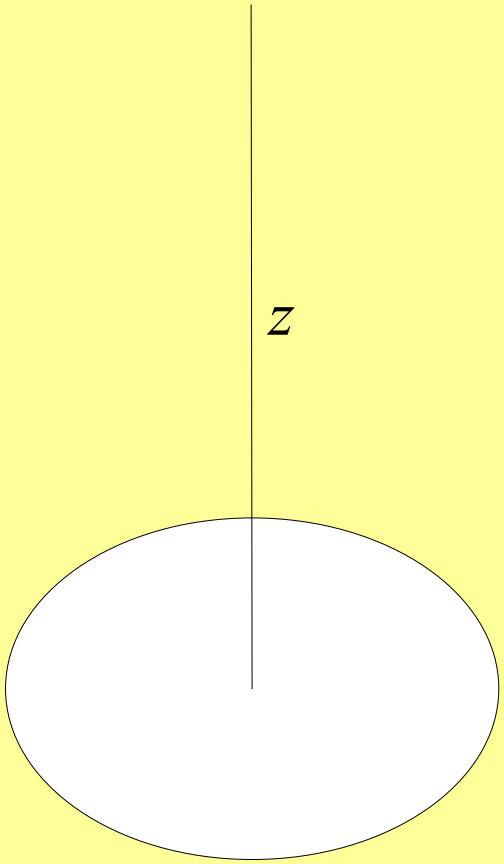


The Content No One Owns

An Example

- Typical of EARLY upper-division work for physics majors and many engineers.
- Solution requires:
  - only one physics concept,
  - only two mathematics skills,
  - many geometrical and visualization strategies.
- Demonstrates different use of language.

Potential Due to Charged Disk



What is the electrostatic potential at a point, on axis, above a uniformly charged disk?

One Physics Concept

- Coulomb's Law:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

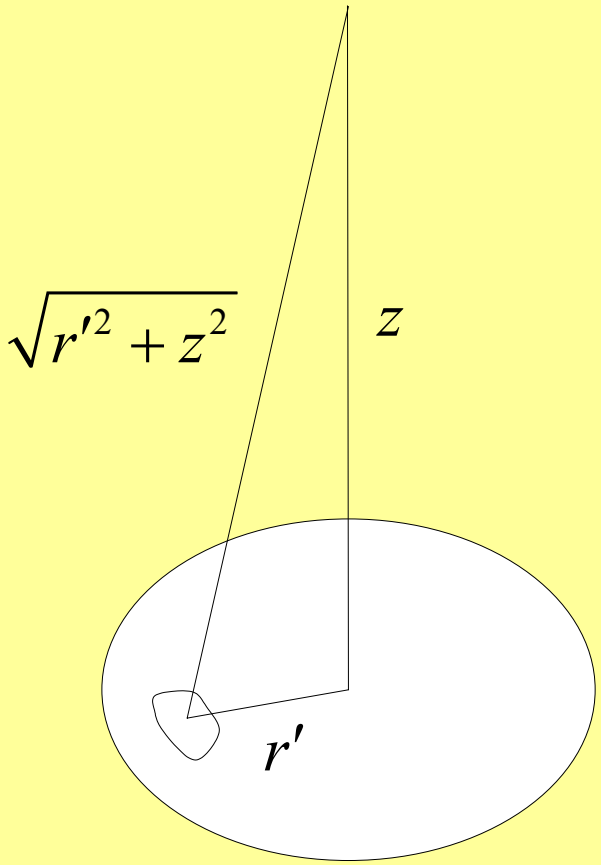
Superposition

- Superposition for solutions of linear differential equations:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\rightarrow V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r}')da'}{\left|\vec{r}-\vec{r}'\right|}$$

Chopping and Adding



Integrals involve chopping up a part of space and adding up a physical quantity on each piece.

Computational Skill

- Can the students set-up and do the integral?

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r}')da'}{\left|\vec{r}-\vec{r}'\right|}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{R} \frac{dr'\,r'\,d\theta'}{\sqrt{r'^2+z^2}}$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)$$

Limits (Far Away)

$$V(\vec{r}) = \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(z\sqrt{1 + \frac{R^2}{z^2}} - z\right)$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(z\left(1 + \frac{1}{2}\frac{R^2}{z^2} + \ldots\right) - z\right)$$

$$\approx \frac{1}{4\pi\varepsilon_0}\frac{\pi R^2\sigma}{z}$$

Importance of Interactive Classroom

Lecture vs. Activities

- The Instructor:
  - Paints big picture.
  - Inspires.
  - Covers lots fast.
  - Models speaking.
  - Models problem-solving.
  - Controls questions.
  - Makes connections.
- The Students:
  - Focus on subtleties.
  - Experience delight.
  - Slow, but in depth.
  - Practice speaking.
  - Practice problem-solving.
  - Control questions.
  - Make connections.

Socratic vs. Groups

How does it feel to teach in these ways?

$$\int_{class} d\ knowledge \quad \text{vs.} \quad \int_{class} d\ questions$$

Everyone knows everything vs. No one knows anything