Bridging the GAP

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Oregon State University

http://www.math.oregonstate.edu/bridge

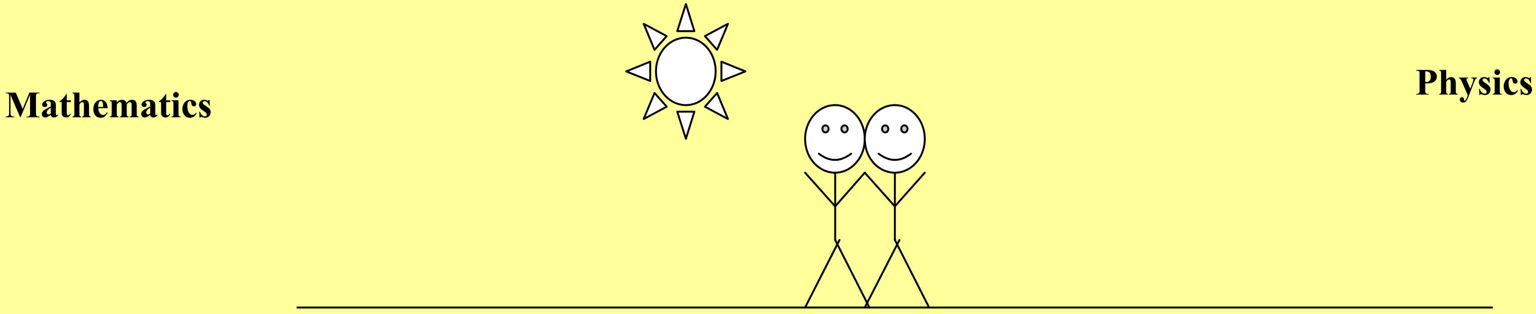
http://www.physics.oregonstate.edu/bridge

http://www.physics.oregonstate.edu/paradigms

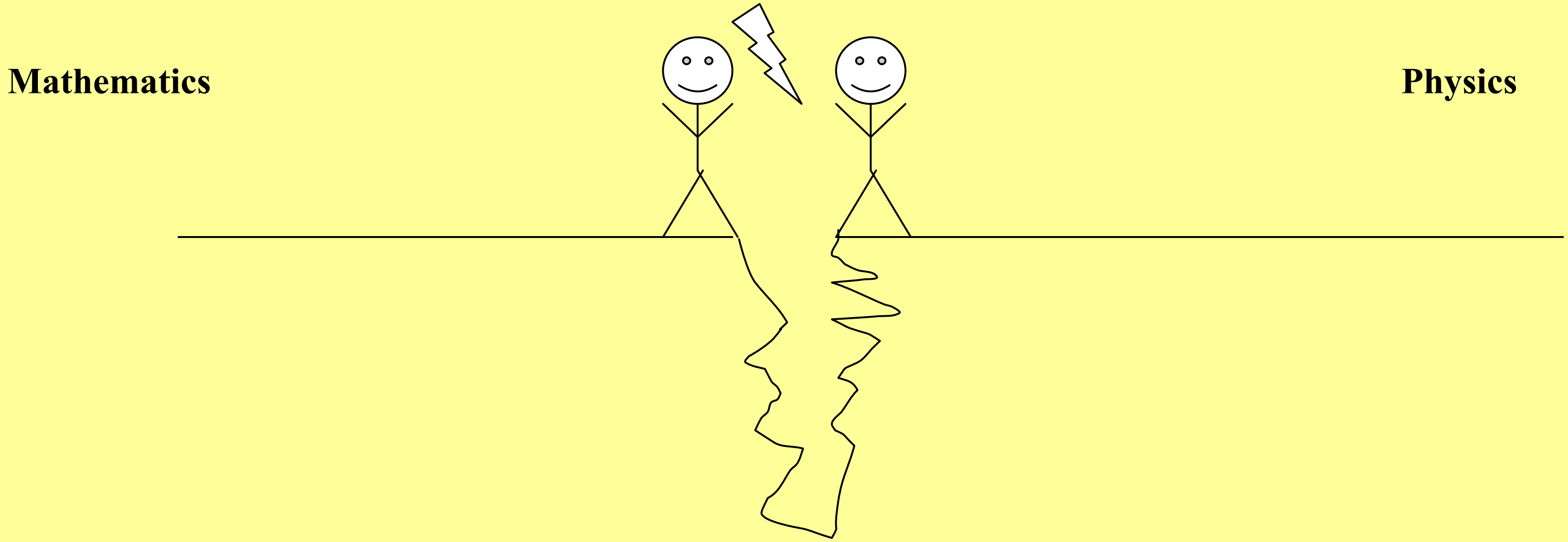
Support

- Oregon State University
  - Department of Mathematics
  - Department of Physics
- Grinnell College
  - Noyce Visiting Professorship
- Mount Holyoke College
  - Hutchcroft Fund
- National Science Foundation
  - DUE-9653250
  - DUE-0231194
  - DUE-0088901
  - DUE-0231032
- Oregon Collaborative for Excellence in the Preparation of Teachers

Mathematics Physics



The Grand Canyon



THE BRIDGE PROJECT

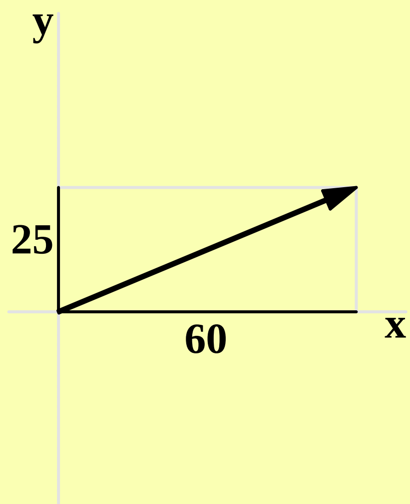
- Small group activities
- Instructor's guide (in preparation)
- CWU, MHC, OSU, UPS, UWEC
- `http://www.physics.orst.edu/bridge`
- Workshops, MAA Minicourse

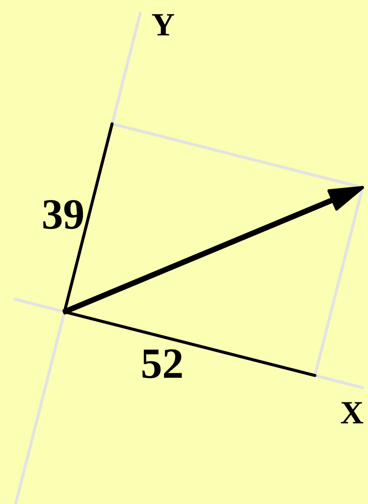
- **Differentials** *(Use what you know!)*
- **Multiple representations**
- **Symmetry** *(adapted bases, coordinates)*
- **Geometry** *(vectors, div, grad, curl)*

CORIANDER

I: In the small town of Coriander, the library can be found by starting at the center of the town square, walking 25 meters north, turning 90° to the right, and walking a further 60 meters.

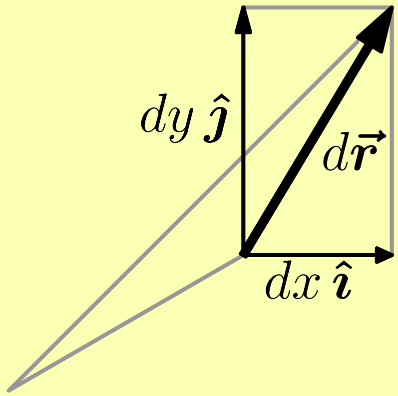
II: It turns out that magnetic north in Coriander is approximately 14° degrees east of true north. If you use a compass to find the library (!), the above directions will fail. Instead, you must walk 39 meters in the direction of magnetic north, turn 90° to the right, and walk a further 52 meters.



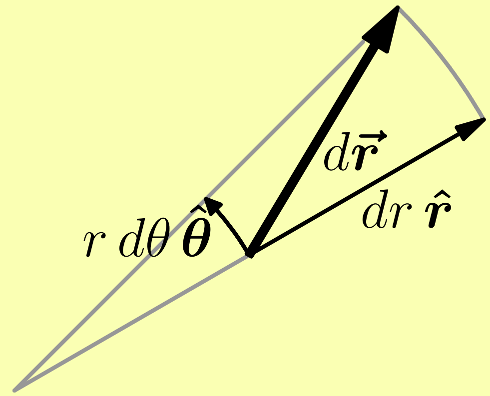


Where on Earth is Coriander?

VECTOR DIFFERENTIALS



$$d\vec{\boldsymbol{r}} = dx\,\hat{\boldsymbol{\imath}} + dy\,\hat{\boldsymbol{\jmath}}$$



$$d\vec{\boldsymbol{r}} = dr\,\hat{\boldsymbol{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}}$$

$$\begin{aligned}
ds &= |d\vec{\boldsymbol{r}}| \\
d\vec{\boldsymbol{S}} &= d\vec{\boldsymbol{r}}_1 \times d\vec{\boldsymbol{r}}_2 \\
dS &= |d\vec{\boldsymbol{r}}_1 \times d\vec{\boldsymbol{r}}_2| \\
dV &= (d\vec{\boldsymbol{r}}_1 \times d\vec{\boldsymbol{r}}_2) \cdot d\vec{\boldsymbol{r}}_3 \\
df &= \vec{\nabla} f \cdot d\vec{\boldsymbol{r}}
\end{aligned}$$

MATHEMATICIANS' LINE INTEGRALS

- Start with Theory

$$\begin{aligned}\int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} &= \int \vec{\boldsymbol{F}} \cdot \hat{\boldsymbol{T}}\, ds \\ &= \int \vec{\boldsymbol{F}}(\vec{\boldsymbol{r}}(t)) \cdot \frac{\vec{\boldsymbol{r}}'(t)}{|\vec{\boldsymbol{r}}'(t)|} |\vec{\boldsymbol{r}}'(t)|\, dt \\ &= \int \vec{\boldsymbol{F}}(\vec{\boldsymbol{r}}(t)) \cdot \vec{\boldsymbol{r}}'(t)\, dt \\ &= \ldots = \int P\, dx + Q\, dy + R\, dz\end{aligned}$$

- Do examples starting from next-to-last line

Need parameterization $\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}(t)$

MATHEMATICS

$$\vec{\boldsymbol{F}}(x,y) = \frac{-y\,\hat{\boldsymbol{\imath}} + x\,\hat{\boldsymbol{\jmath}}}{x^2+y^2} \qquad \vec{\boldsymbol{r}} = x\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}}$$

$$\begin{aligned} x &= 2\cos\theta \\ y &= 2\sin\theta \end{aligned}$$

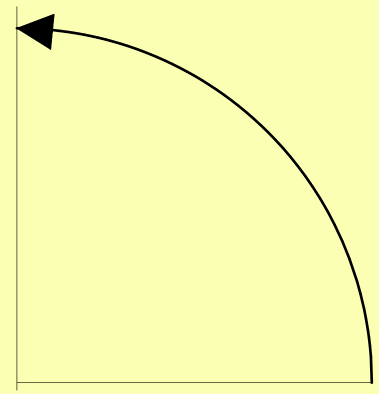
$$\begin{aligned} \int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} &= \int_0^{\frac{\pi}{2}} \vec{\boldsymbol{F}}(x(\theta), y(\theta)) \cdot \vec{\boldsymbol{r}}\,'(x(\theta), y(\theta))\, d\theta \\ &= \int_0^{\frac{\pi}{2}} \frac{1}{2}(-\sin\theta\,\hat{\boldsymbol{\imath}} + \cos\theta\,\hat{\boldsymbol{\jmath}}) \cdot 2(-\sin\theta\,\hat{\boldsymbol{\imath}} + \cos\theta\,\hat{\boldsymbol{\jmath}})\, d\theta \\ &= \ldots = \frac{\pi}{2} \end{aligned}$$

$\boxed{\textbf{PHYSICISTS' LINE INTEGRALS}}$

- Theory
  - Chop up curve into little pieces $d\vec{\boldsymbol{r}}$.
  - Add up components of $\vec{\boldsymbol{F}}$ parallel to curve (times length of $d\vec{\boldsymbol{r}}$)
- Do examples directly from $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$

$$\boxed{\text{Need } d\vec{\boldsymbol{r}} \text{ along curve}}$$

PHYSICS



$$\vec{\boldsymbol{F}} = \frac{\hat{\boldsymbol{\theta}}}{r}$$

$$d\vec{\boldsymbol{r}} = r\,d\theta\,\hat{\boldsymbol{\theta}}$$

I: $|\vec{\boldsymbol{F}}| = \text{const};\ \vec{\boldsymbol{F}} \parallel d\vec{\boldsymbol{r}} \Longrightarrow$

$$\int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = \frac{1}{2}(2\ \frac{\pi}{2})$$

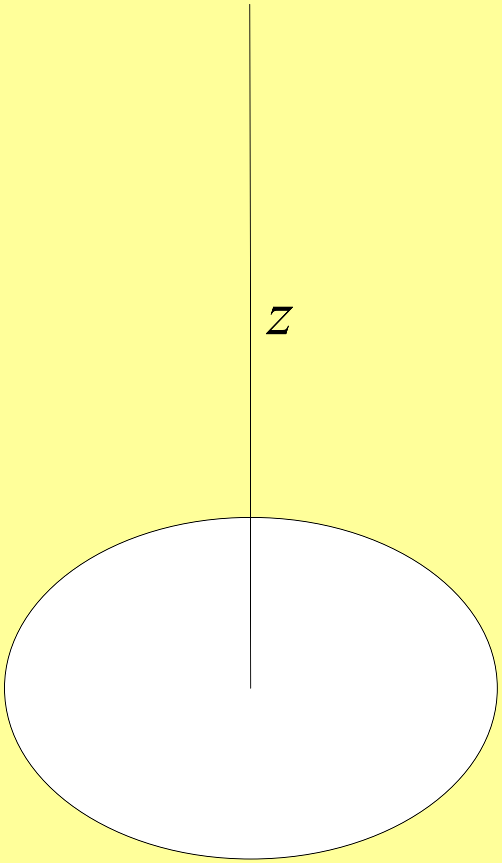
II: Do the dot product $\longmapsto$

$$\int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = \int_0^{\frac{\pi}{2}} \frac{\hat{\boldsymbol{\theta}}}{2} \cdot 2\,d\theta\,\hat{\boldsymbol{\theta}} = \int_0^{\frac{\pi}{2}} d\theta = \frac{\pi}{2}$$

An Example

- Typical of EARLY upper-division work for physics majors and many engineers.
- Solution requires:
  - many mathematical strategies,
  - many geometrical and visualization strategies,
  - only one physics concept.
- Demonstrates different use of language.

Potential Due to Charged Disk



What is the electrostatic potential at a point, on axis, above a uniformly charged disk?

One Physics Concept

- Coulomb's Law:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

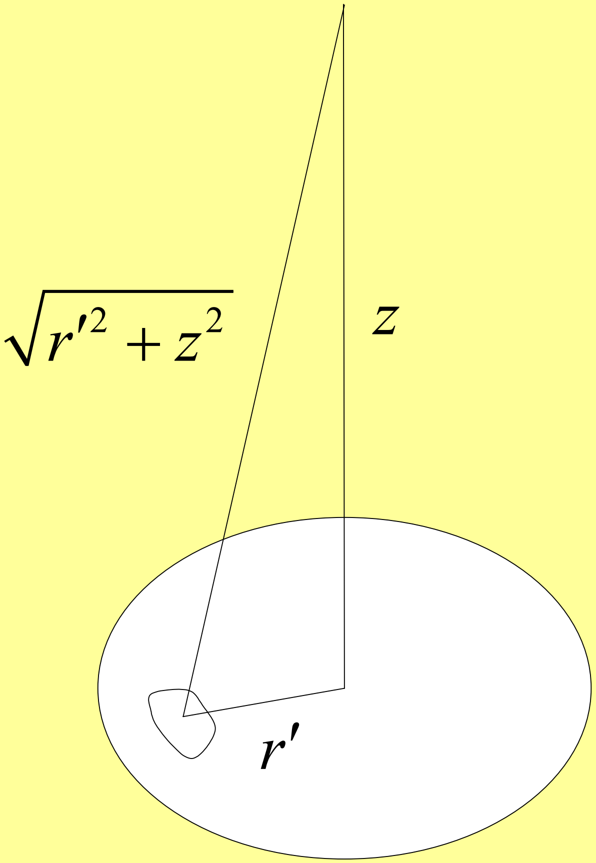
Superposition

- Superposition for solutions of linear differential equations:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\rightarrow V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r}')da'}{\left|\vec{r}-\vec{r}'\right|}$$

Chopping and Adding



Integrals involve chopping up a part of space and adding up a physical quantity on each piece.

Computational Skill

- Can the students set-up and do the integral?

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r}')da'}{|\vec{r}-\vec{r}'|}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{R} \frac{dr'\,r'\,d\theta'}{\sqrt{r'^2+z^2}}$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)$$

Limits (Far Away)

$$V(\vec{r}) = \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(z\sqrt{1 + \frac{R^2}{z^2}} - z\right)$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\left(z\left(1 + \frac{1}{2}\frac{R^2}{z^2} + \ldots\right) - z\right)$$

$$\approx \frac{1}{4\pi\varepsilon_0}\frac{\pi R^2\sigma}{z}$$

Simple Curriculum Additions

- Equations describe the relationship of physical quantities to other physical quantities.
- Emphasize superposition of solutions to linear differential equations.
- Integrals involve chopping a part of space and adding up a physical quantity on each piece.
- “Limits” require one to say what dimensionless quantity is small.

Opportunities

Beta testers are being sought to test materials:

- Vector Calculus (lower-division math)
  http://www.physics.oregonstate.edu/bridge
  - MathFest (8/12-14/04)
  - Summer Workshops (6/18-22/04 MA) (8/7-11/04 OR)
- Thermodynamics and Quantum Mechanics
  http://www.physics.oregonstate.edu/paradigms
  - Summer Workshop (6/25-28/04)
  - AAPT Workshop (1/25/04)