Let's Do This Once

Folding Vector Calculus into Multivariable Calculus Using Geometry and Language

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Two Courses Filled Full of New Ideas for Students

Calculus III	Vector Calculus
$\mathbf{R}^3$, rect., cyl., spher. coord.	Vector Fields
Vector computations	Div, Grad, Curl
Double and triple integrals	Fund. Thm. of Line Integrals
Partial derivatives	Stokes' Theorem
Directional Derivatives	Divergence Theorem
Optimization Problems	Conservative Vector Fields
Recipes for computations...	Students build on concepts??

How can we help students learn concepts in Vector Calculus?

The Vector Calculus Bridge Project
Tevian Dray and Corinne Manogue, Oregon State University

Emphasize Geometry!

Alternate approach from Calculus III skills:

1. Emphasize geometry of vector operations
   - Dot product is a (weighted) projection
   - Cross product is a directed area
2. Emphasize differentials and rates of change
3. Emphasize vector differentials
4. Emphasize coordinate-independent formulas

Result: Students pull correct equations from the geometry of a problem and use it to correctly find solutions.

The Problem: Fold these two courses into one course

- Goal 1: Cover Calculus III ideas and techniques
- Goal 2: Cover Vector Calculus theorems
- Goal 3: Few recipes... Students understand it all!

Is this really possible in 16 weeks?

For Goal 3, want Bridge Project approach...

How about in 13 weeks???

Emphasize Many Forms of Differentiation

For a function f and vector field $\vec{G}$, we utilize the following derivatives:

- Differentials df
- Vector Differentials $d\vec{r}$
- Gradient Vector $\vec{\nabla} f$
- Divergence of Vector Field: $\vec{\nabla} \cdot \vec{G}$
- Curl of Vector Field: $\vec{\nabla} \times \vec{G}$
- *Directional Derivative $\vec{\nabla} f \cdot \hat{u}$
- *Mixed partial derivatives

(* get for free)

Solving Derivative Problems with Language

In class, emphasize Linearization and Geometry

- Ex. 1: If your position is $\vec{r}$, then $d\vec{r}$ is the change in position.
- Ex. 2: $\vec{\nabla} f$ describes how f is changing. $d\vec{r}$ describes how a path p is changing. So, $\vec{\nabla} f \cdot d\vec{r}$ describes how f changes along p.
- Implications:
  $\vec{\nabla} f \cdot \hat{u}$ describes a directional derivative.
  $\vec{\nabla} f \cdot d\vec{r} = 0$ is method of Lagrange Multipliers.

Emphasize Tool of Integration

Integrals are used to give total amount.

- Ex 1. $\vec{\nabla} f \cdot d\vec{r}$ describes how f changes along p.
  So, $\int \vec{\nabla} f \cdot d\vec{r}$ is total change along path p.

Let $\{p\}$ and $\{q\}$ be two families of level curves covering a surfac
Let $d\vec{r_p}$ and $d\vec{r_q}$ describe change along curves.
$d\vec{A} = d\vec{r_p} \times d\vec{r_q}$ is small (directed) area.

- Ex 2. $\int |d\vec{A}|$ is total surface area bounded by curves.
  $\vec{G} \cdot d\vec{A}$ is flux of $\vec{G}$ through $d\vec{A}$.
  $\int \vec{G} \cdot d\vec{A}$ is total flux.

The Fundamental Theorem

Students work with a singular view of the major theorems:

Fundamental Theorem of Line Integrals
$\int_{\partial[a,b]} f = f(b) - f(a) = \int_a^b \vec{\nabla} f \cdot d\vec{r}$

Divergence Theorem
$\int_{\partial R} \vec{G} \cdot d\vec{A} = \int_R \vec{\nabla} \cdot \vec{G} dV$

Stokes' Theorem
$\int_{\partial R} \vec{G} \cdot d\vec{r} = \int_R \vec{\nabla} \times \vec{G} \cdot d\vec{A}$

"Haven't we been doing the same thing all semester?"

Summary

- Emphasize basic geometric concepts from Bridge Project.
- Learn calculations after concepts and language.
- Cover more content in a streamlined fashion. Students forced to work on concepts rather than recipes.

Thank you! awangberg@winona.edu