Geometric Visualization as the Bridge between Math and Applications

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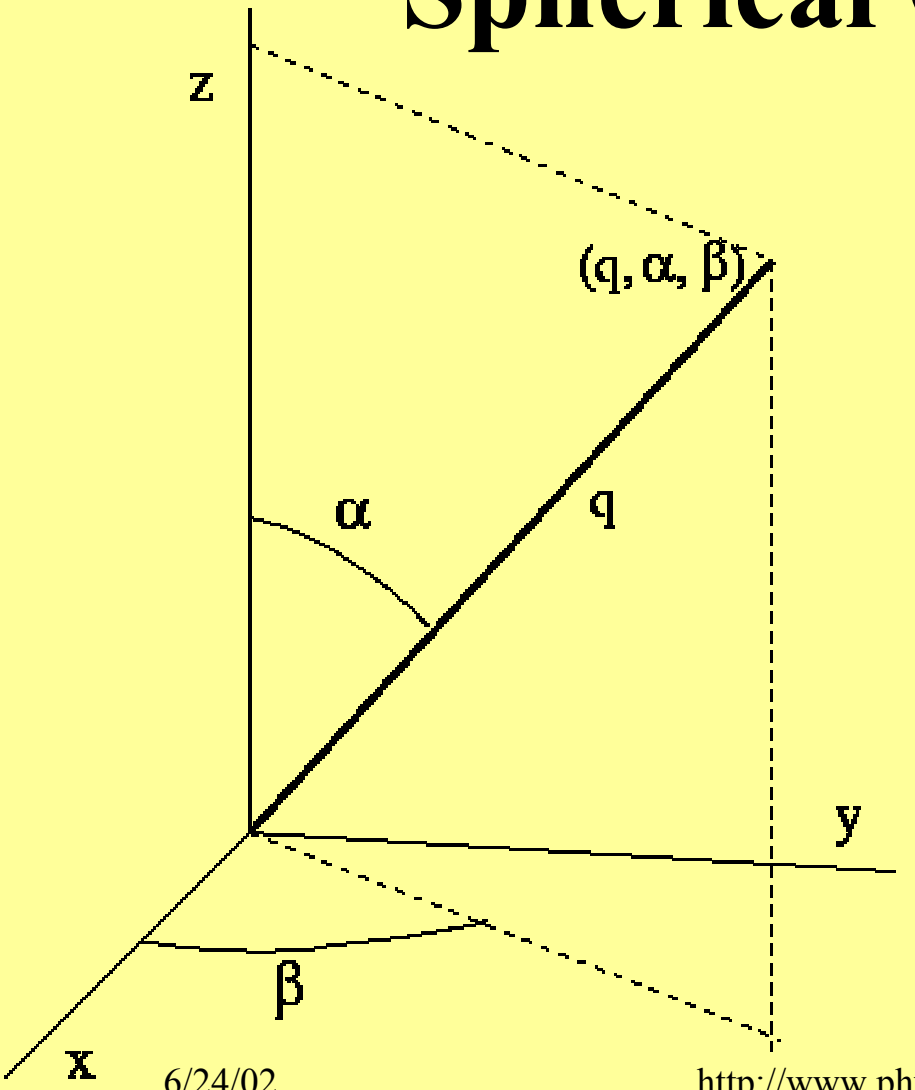
Spherical Coordinates



q: Radius

α: Zenith (colatitude)

β: Azimuth (longitude)

Math: (ρ, ϕ, θ)

Physics: (r, θ, ϕ)

Functions

Suppose the temperature on a rectangular slab of metal is given by:

$$T(x,y) = k(x^2 + y^2)$$

Physics: $T(r,\theta) = kr^2$

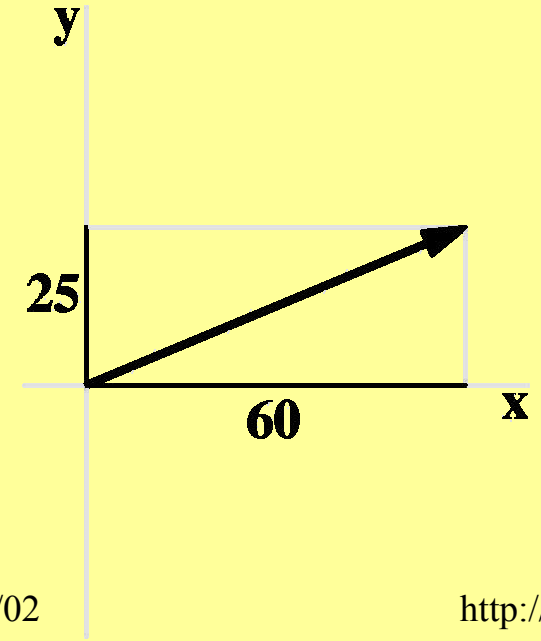
Math: $T(r,\theta) = k(r^2 + \theta^2)$

What Are Vectors?

Mathematics:

- Triples of numbers

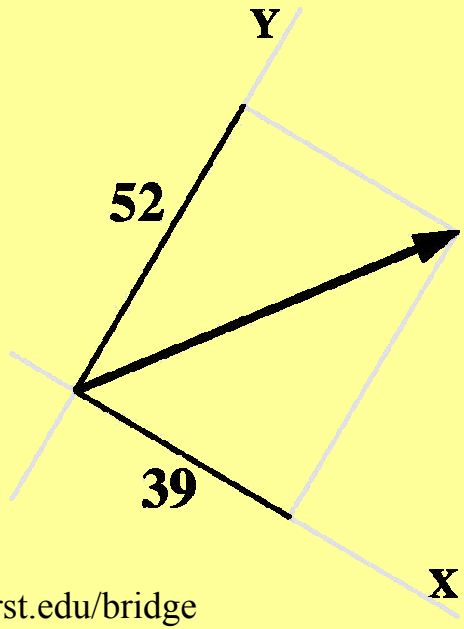
$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$



Physics:

- Arrows in space

$$\vec{v} = v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}$$



Do You Do This?

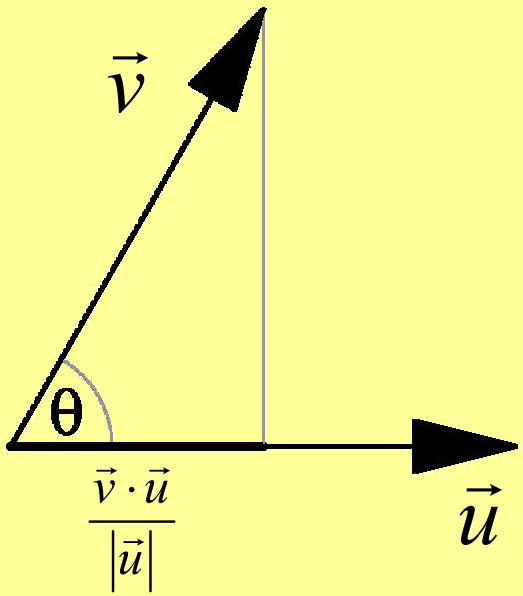
$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$\left|\vec{u}\right| = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$$

Or This?

$$\vec{u} \cdot \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

$$\left|\vec{u}\right| = \sqrt{\vec{u} \cdot \vec{u}}$$



Group Activities

- Task Master

  Keeps group on track:

  "What you had for lunch doesn't seem relevant."

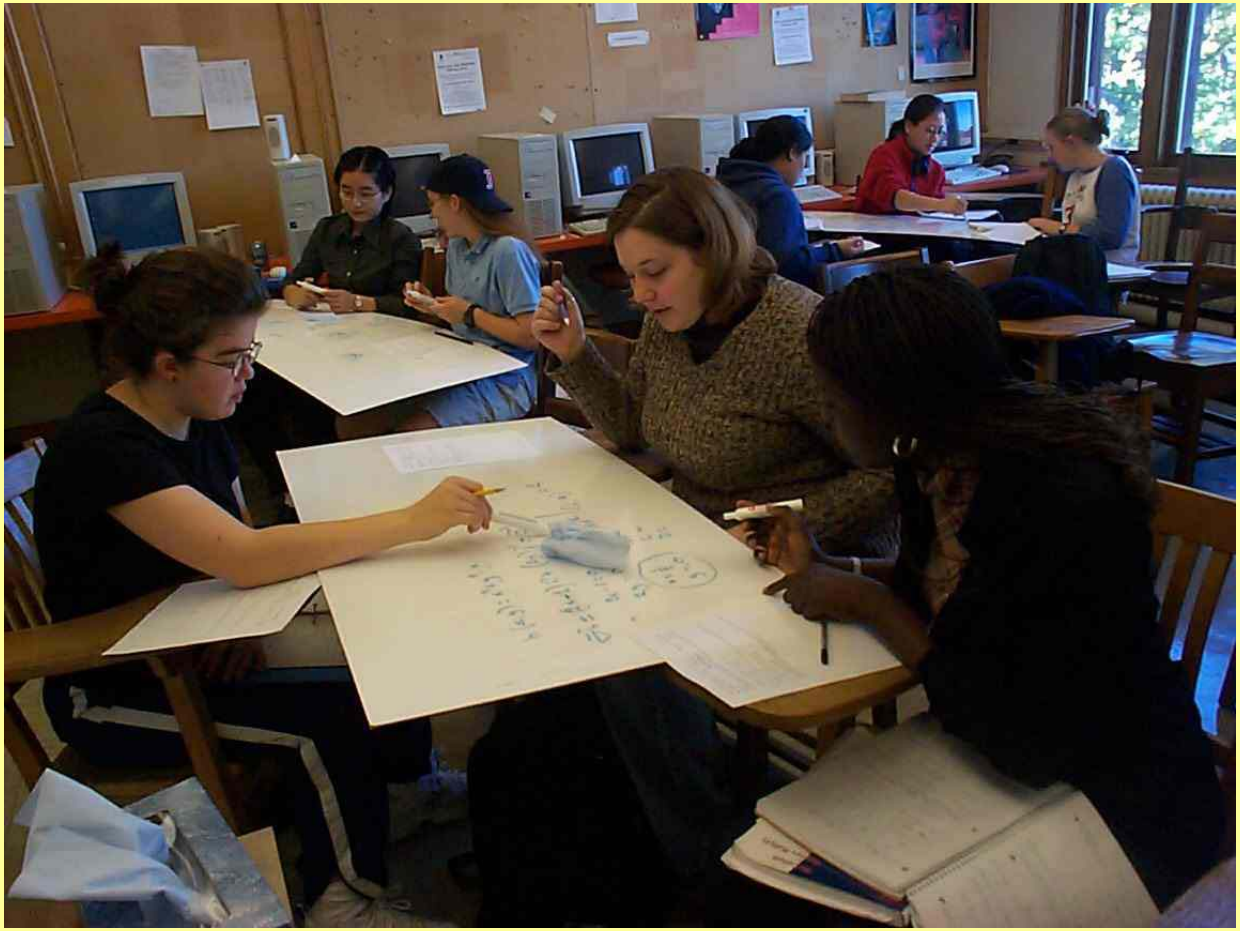
- Cynic

  Questions everything:

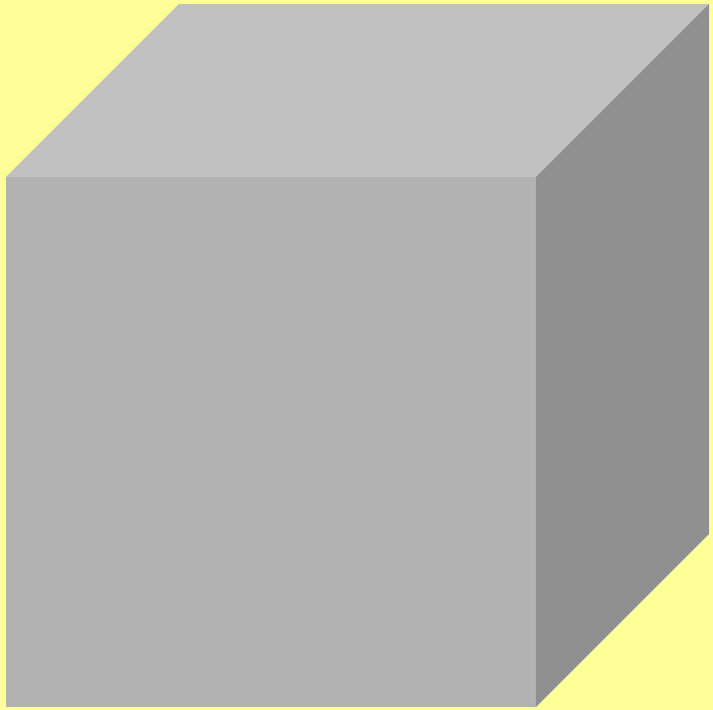
  "Why?" "Why?" "Why?"

- Recorder
- Reporter

Group Activities



Sample Group Activity



Find the angle between a diagonal of a cube and an edge

Group Activity

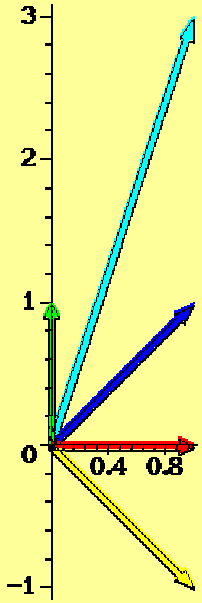
- Emphasizes that vectors are arrows
- Combines geometry and algebra
- Uses multiple representations

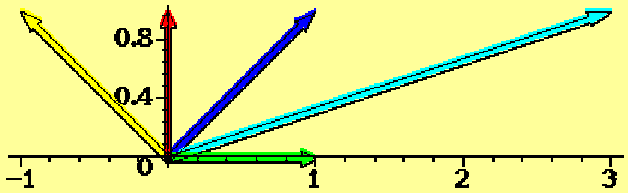
geometry: $\vec{u} \cdot \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right| \cos\theta$

algebra: $\vec{u} \cdot \vec{v} = \left(u_1 \hat{\imath} + u_2 \hat{\jmath} + u_3 \hat{k}\right) \cdot \left(v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}\right)$

memory: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Eigenvectors Activity





- Draw the initial vectors below on a single graph
- Operate on the initial vectors with your group's matrix and graph the transformed vectors

Eigenvectors Activity

- Note any differences between the initial and transformed vectors. Are there any vectors which are left unchanged by your transformation?
- Sketch your transformed vectors on the chalkboard.

The Bridge Project

- Differentials (Use what you know!)
- Exploiting multiple representations
- Symmetry (adapted bases, coordinates)
- Geometry (vectors, div, grad, curl)
- Computer visualization (Maple, Excel)
- Active engagement (groups, flashcards)

Differentials

$$df = \frac{df}{dx}dx$$

Substitution

$$\int 2x \sin x \, dx$$

$$u = x^2$$

$$du = 2x\,dx$$

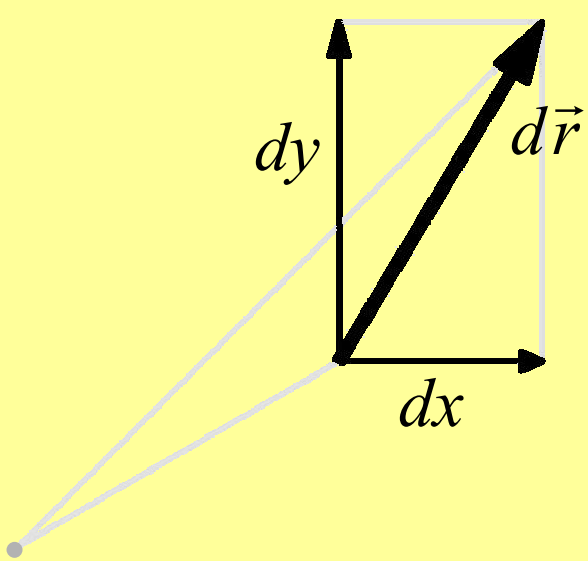
Chain Rule

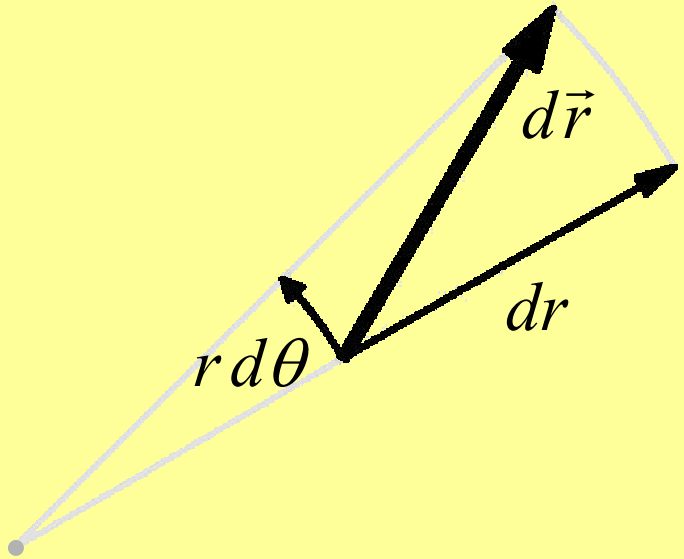
$$x = \cos\theta$$

$$dx = -\sin\theta\, d\theta$$

$$du = 2x\,dx = 2\cos\theta(-\sin\theta\, d\theta)$$

Vector Differentials





$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

Early Problems

- The “gap” is bigger than expected
- Difficulty relating labs to lectures
- Insufficient emphasis on homework
- Open-ended problems make planning hard
- Some students don’t like small groups
- Keeping groups on track

Early Successes

- Student enthusiasm – many lightbulbs!
- Students use geometric reasoning
- Conceptual understanding appears better
- Some at-risk students blossom
- Student camaraderie

Current Status

- Proof of Concept *(2001/2002)*
  - Labs developed and tried
  - Instructors' Guide in progress
- "Alpha" testing *(2002)*
  - Other instructors *(OSU/MHC)*
- Beta testing *(2003 & beyond)*
  - Other institutions *(contact us if interested!)*

A Radical View of Calculus

- The central idea of calculus is not the limit.
- The central idea of derivatives is not slope.
- The central idea of integrals is not area.
- The central idea of curves and surfaces is not parameterization.
- The central representation of a function is not its graph.