Bridging The Vector Calculus G A P

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Do You Do This?

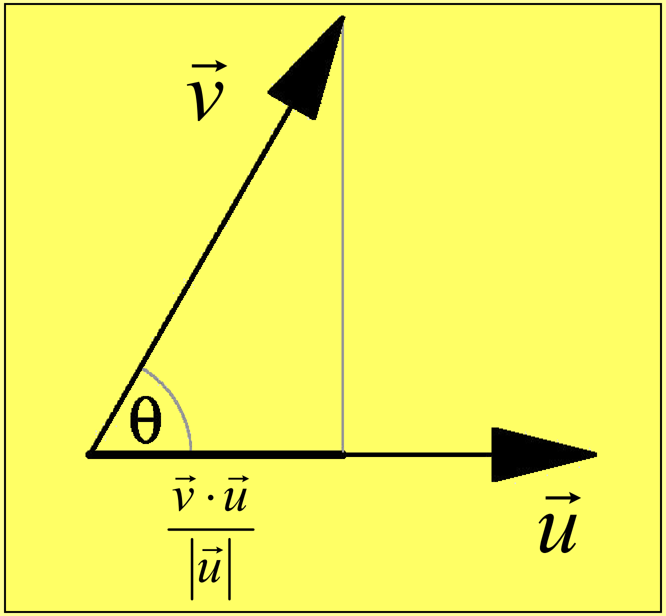
$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

$$\left|\vec{u}\right| = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$$

Or This?

$$\vec{u}\cdot\vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

$$\left|\vec{u}\right| = \sqrt{\vec{u}\cdot\vec{u}}$$



Group Activities

- **Task Master**

  Keeps group on track:

  "What you had for lunch doesn't seem relevant."

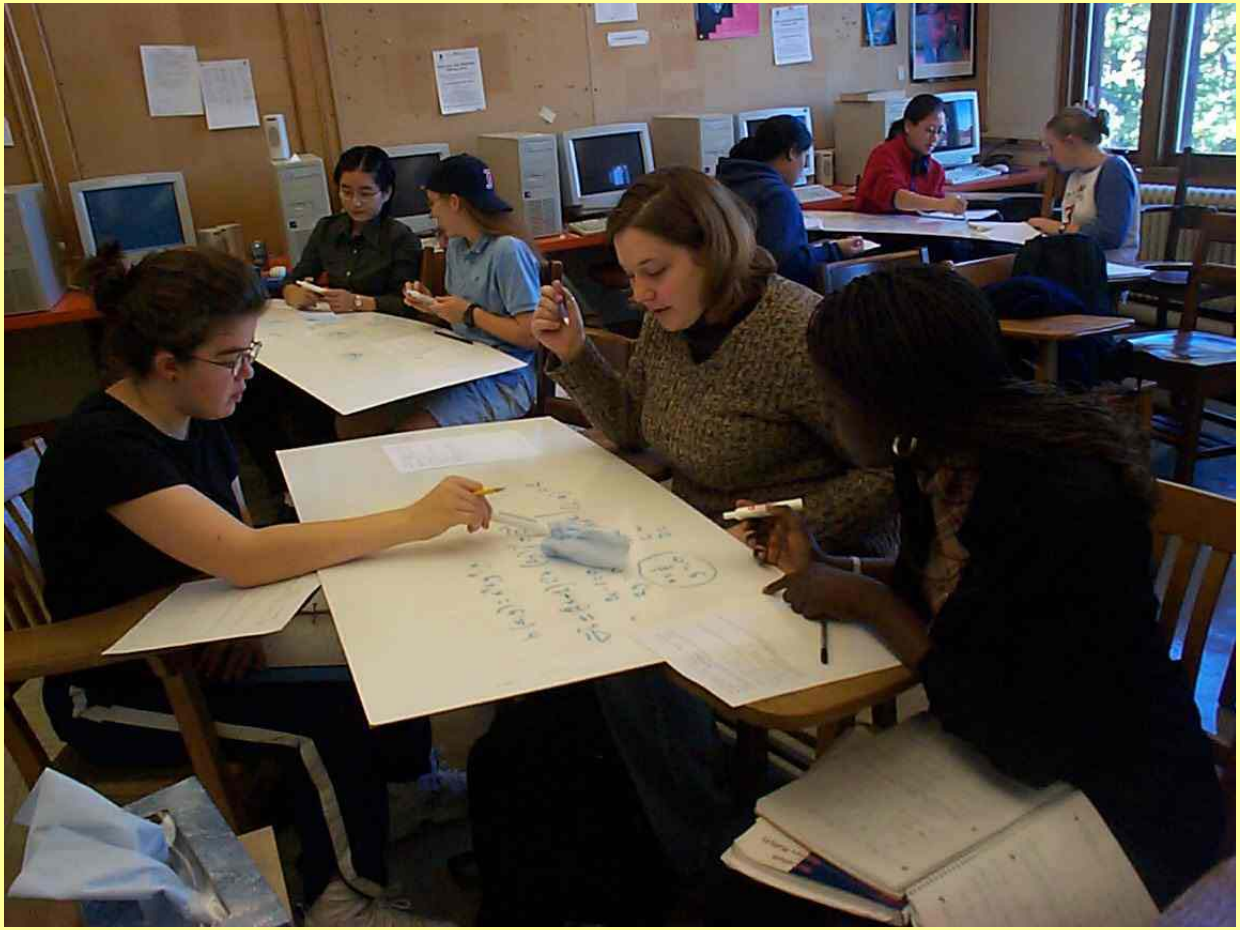
- **Cynic**

  Questions everything:

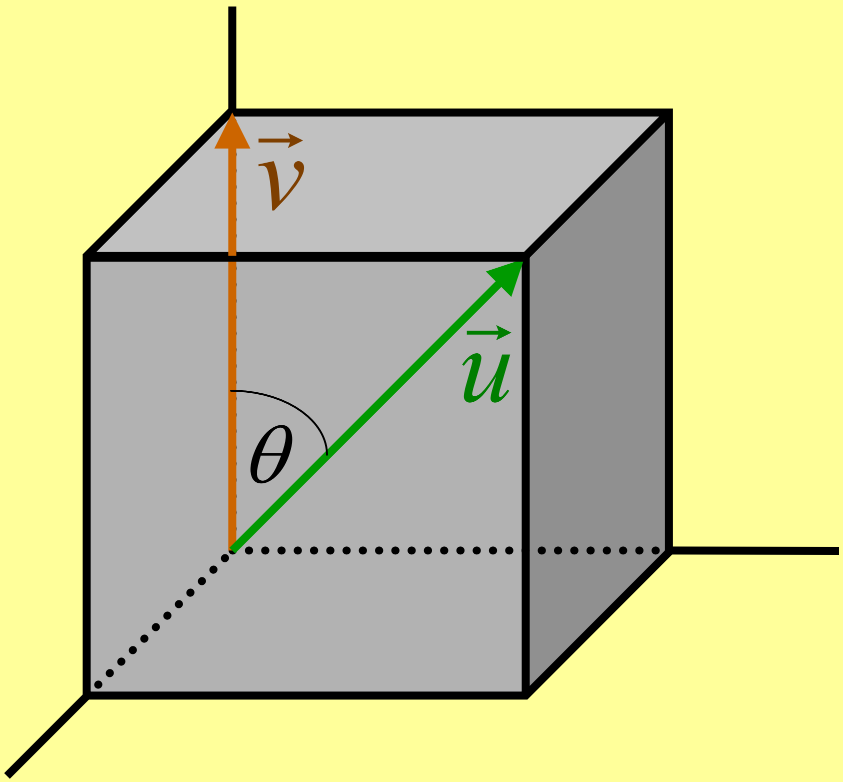
  "Why?" "Why?" "Why?"

- **Recorder**
- **Reporter**

Group Activities



Sample Group Activity



$$\vec{u} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{v} = \hat{k}$$

$$\cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{1}{\sqrt{3}}$$

Find the angle between a diagonal of a cube and an edge

Group Activity

- **Emphasizes that vectors are arrows**
- **Combines geometry and algebra**
- **Uses multiple representations**

geometry: $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

algebra: $\vec{u} \cdot \vec{v} = \left(u_1 \hat{\imath} + u_2 \hat{\jmath} + u_3 \hat{k}\right) \cdot \left(v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}\right)$

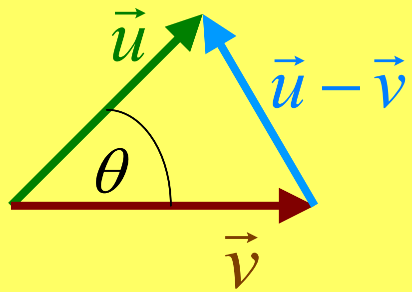
memory: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Start with Algebra?

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

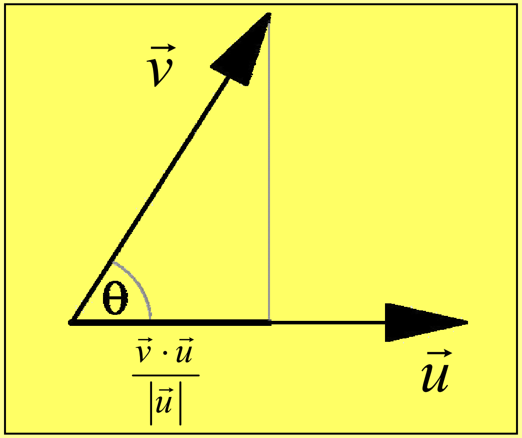
$$\Rightarrow (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} + \vec{v}\cdot\vec{v} - 2\vec{u}\cdot\vec{v}$$

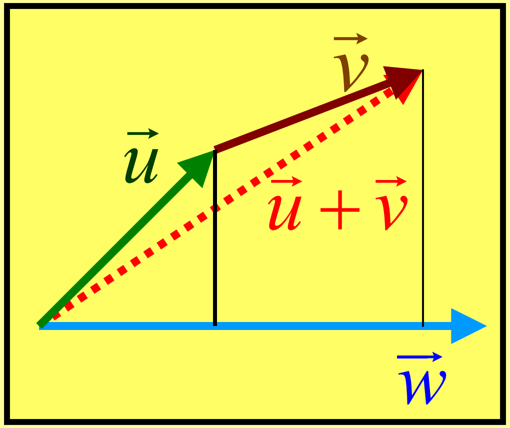
$$\left|\vec{u}-\vec{v}\right|^2 = \left|\vec{u}\right|^2 + \left|\vec{v}\right|^2 - 2\left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$



(Law of Cosines)

Start with Geometry!



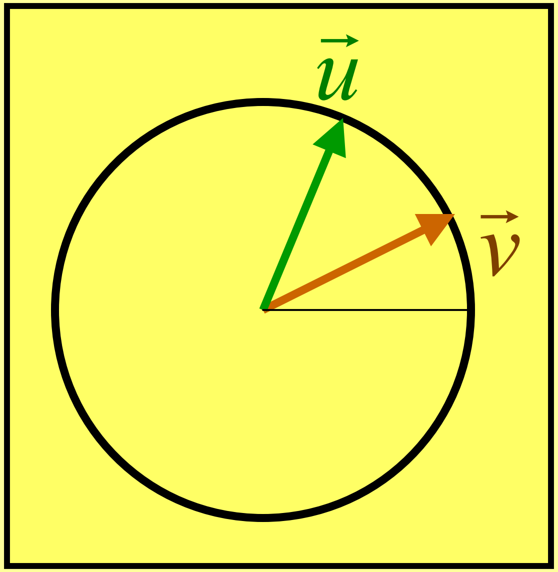


$$\Rightarrow (\vec{u} + \vec{v}) \cdot \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w}$$

$$\Rightarrow \left(u_1 \hat{\imath} + u_2 \hat{\jmath}\right) \cdot \left(v_1 \hat{\imath} + v_2 \hat{\jmath}\right) = u_1 v_1 + u_2 v_2$$

(get Law of Cosines for free!)

Use both!



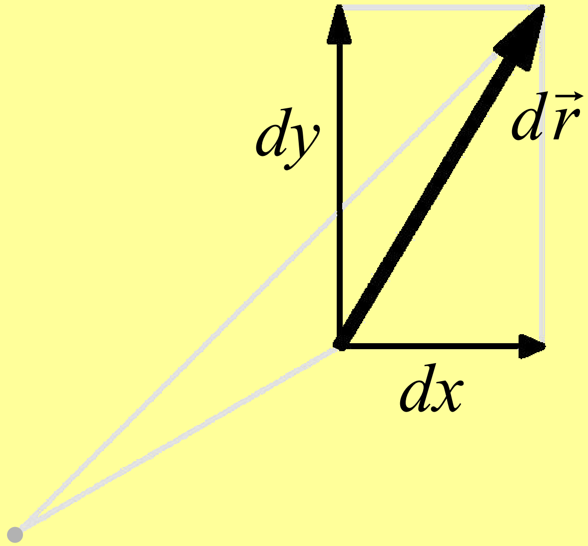
$$\vec{u} = \cos(\alpha)\,\hat{\imath} + \sin(\alpha)\,\hat{\jmath}$$

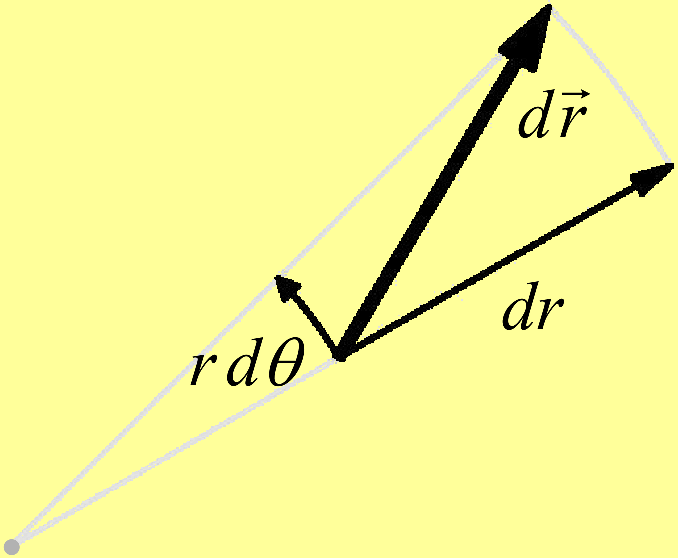
$$\vec{v} = \cos(\beta)\,\hat{\imath} + \sin(\beta)\,\hat{\jmath}$$

$$\vec{u} \cdot \vec{v} = \cos(\alpha - \beta)$$

(get addition formulas for free!)

Vector Differentials





$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

Vector Differentials

- **Line Integrals**
  - $\int \vec{F} \cdot d\vec{r}$
  - $\int f \, ds \qquad ds = \left| d\vec{r} \right|$
- **Surface Integrals**
  - $\int\int \vec{F} \cdot d\vec{S} \qquad d\vec{S} = d\vec{r}_1 \times d\vec{r}_2$
  - $\int\int f \, dS \qquad dS = \left| d\vec{r}_1 \times d\vec{r}_2 \right|$
- **Volume Integrals**
  - $dV = (d\vec{r}_1 \times d\vec{r}_2) \cdot d\vec{r}_3$
- **Gradient**
  - $df = \vec{\nabla} f \cdot d\vec{r}$

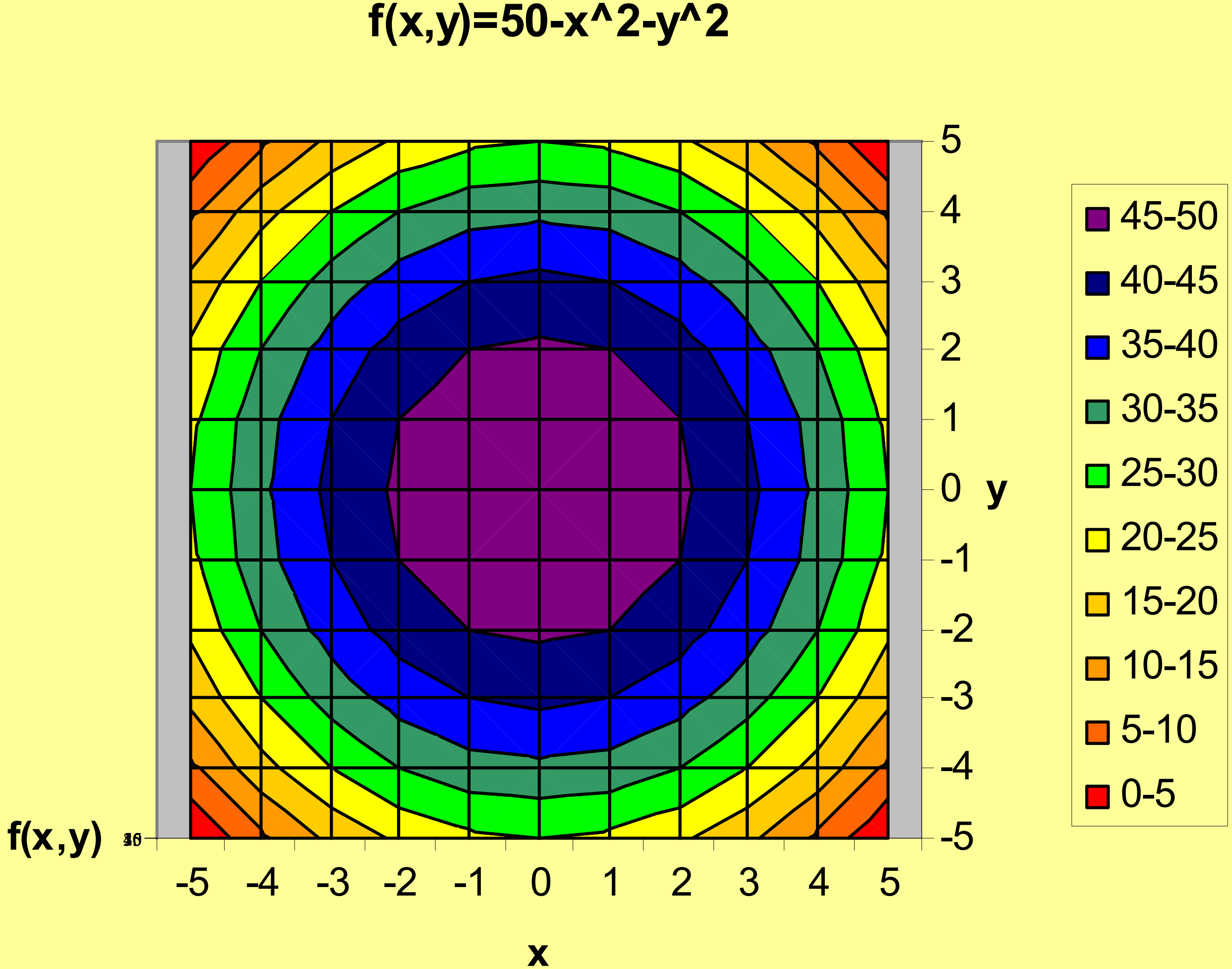
The Bridge Project

- **Differentials (Use what you know!)**
- **Exploiting multiple representations**
- **Symmetry (adapted bases, coordinates)**
- **Geometry (vectors, div, grad, curl)**
- **Computer visualization (Maple, Excel)**
- **Active engagement (groups, flashcards)**

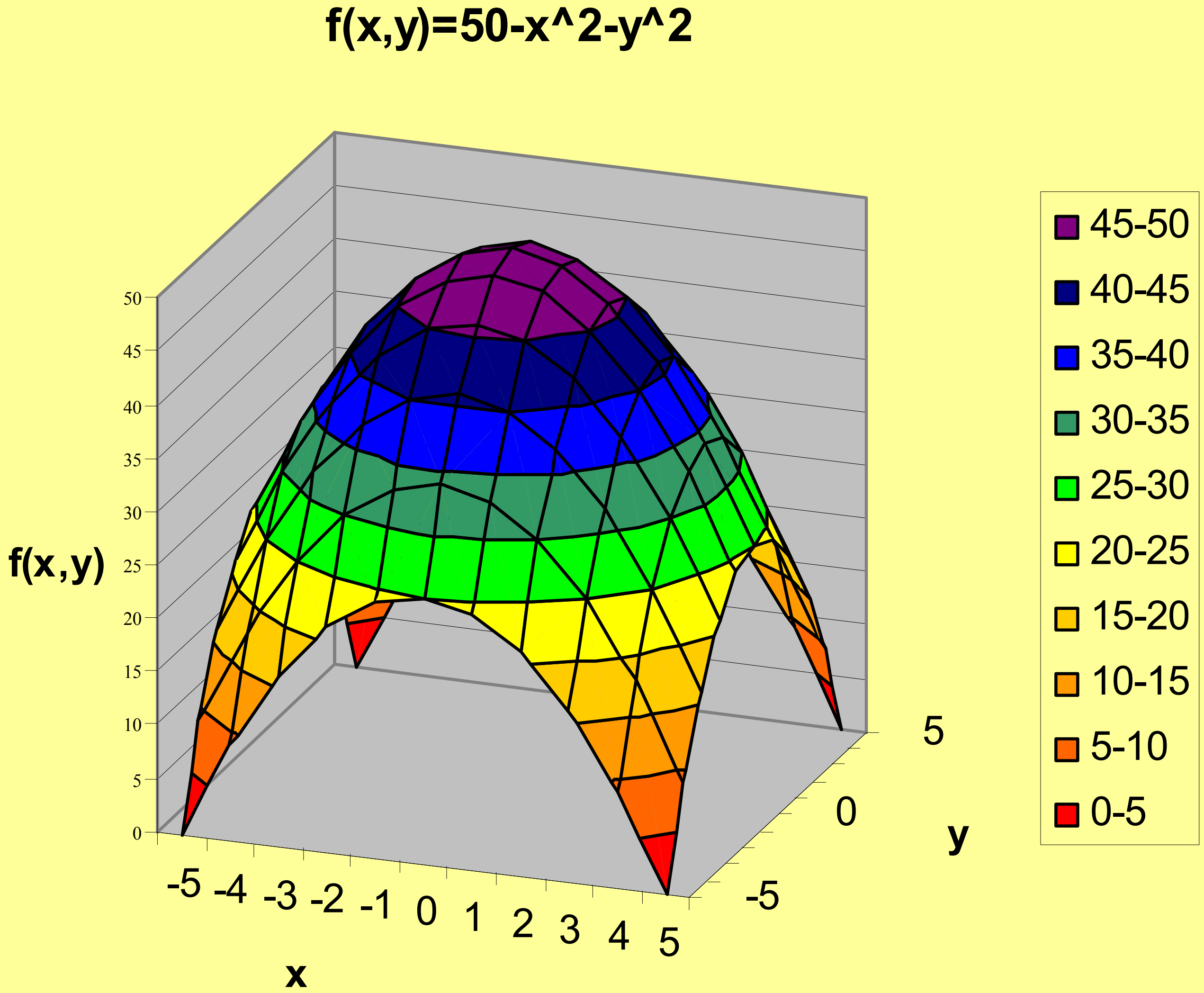
Visualization—Table

	-5	-4	-3	-2	-1	0	1	2	3	4	5
-5	0	9	16	21	24	25	24	21	16	9	0
-4	9	18	25	30	33	34	33	30	25	18	9
-3	16	25	32	37	40	41	40	37	32	25	16
-2	21	30	37	42	45	46	45	42	37	30	21
-1	24	33	40	45	48	49	48	45	40	33	24
0	25	34	41	46	49	50	49	46	41	34	25
1	24	33	40	45	48	49	48	45	40	33	24
2	21	30	37	42	45	46	45	42	37	30	21
3	16	25	32	37	40	41	40	37	32	25	16
4	9	18	25	30	33	34	33	30	25	18	9
5	0	9	16	21	24	25	24	21	16	9	0

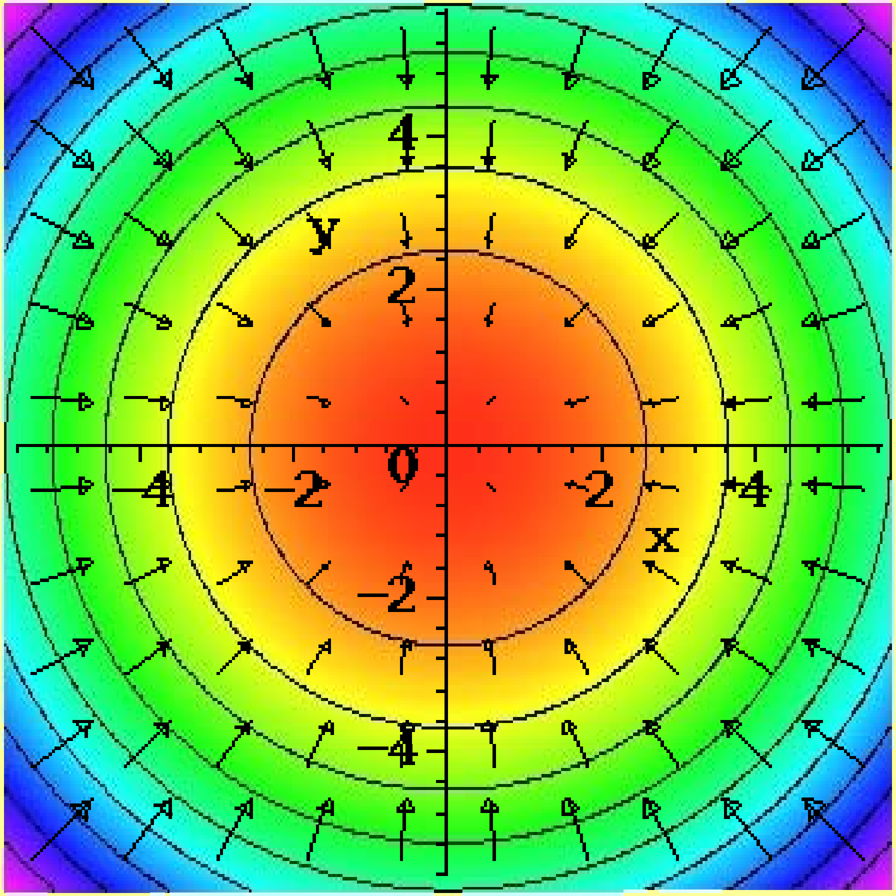
Visualization—Level Curves



Visualization—Graph



Visualization—Gradient



Advice to Curriculum Developers

- The problem is often not what you think—listen to the students.
- Sometimes solving a small early problem solves lots of bigger later problems.
- Don’t be afraid of confusion in the classroom.

A Radical View of Calculus

- The central idea of calculus is not the limit.
- The central idea of derivatives is not slope.
- The central idea of integrals is not area.
- The central idea of curves and surfaces is not parameterization.
- The central representation of a function is not its graph.