Bridging

The Vector Calculus G A P

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Why Study Vector Calculus?

Mathematics:

- Visualization
- Geometric Reasoning
- Multiple Representations
- Reinforce Calculus Ideas
- Differential Geometry
- Complex Variables

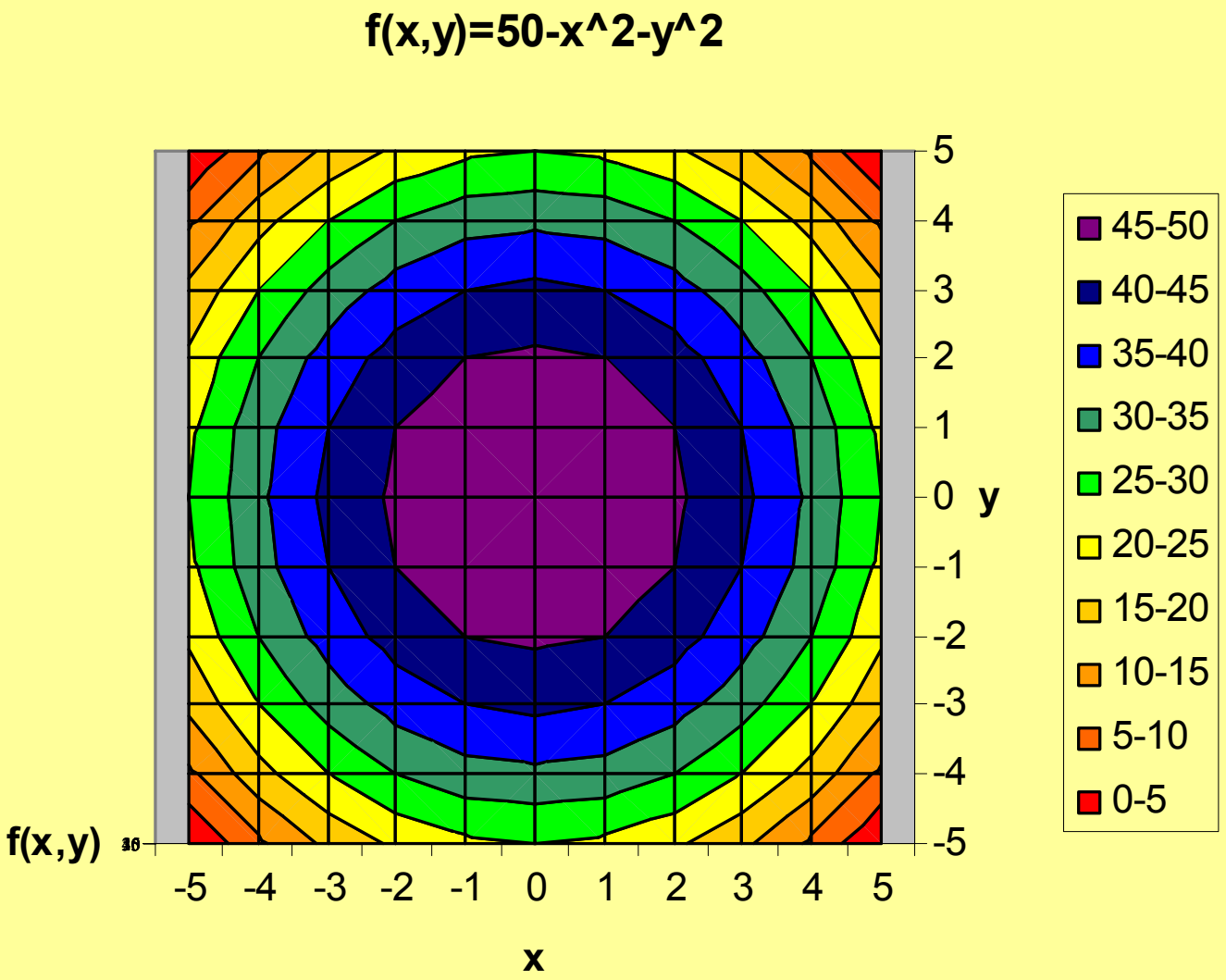
Physics:

- Visualization
- Geometric Reasoning
- Multiple Representations
- Symmetry
- Electromagnetism!!
- Classical Mechanics

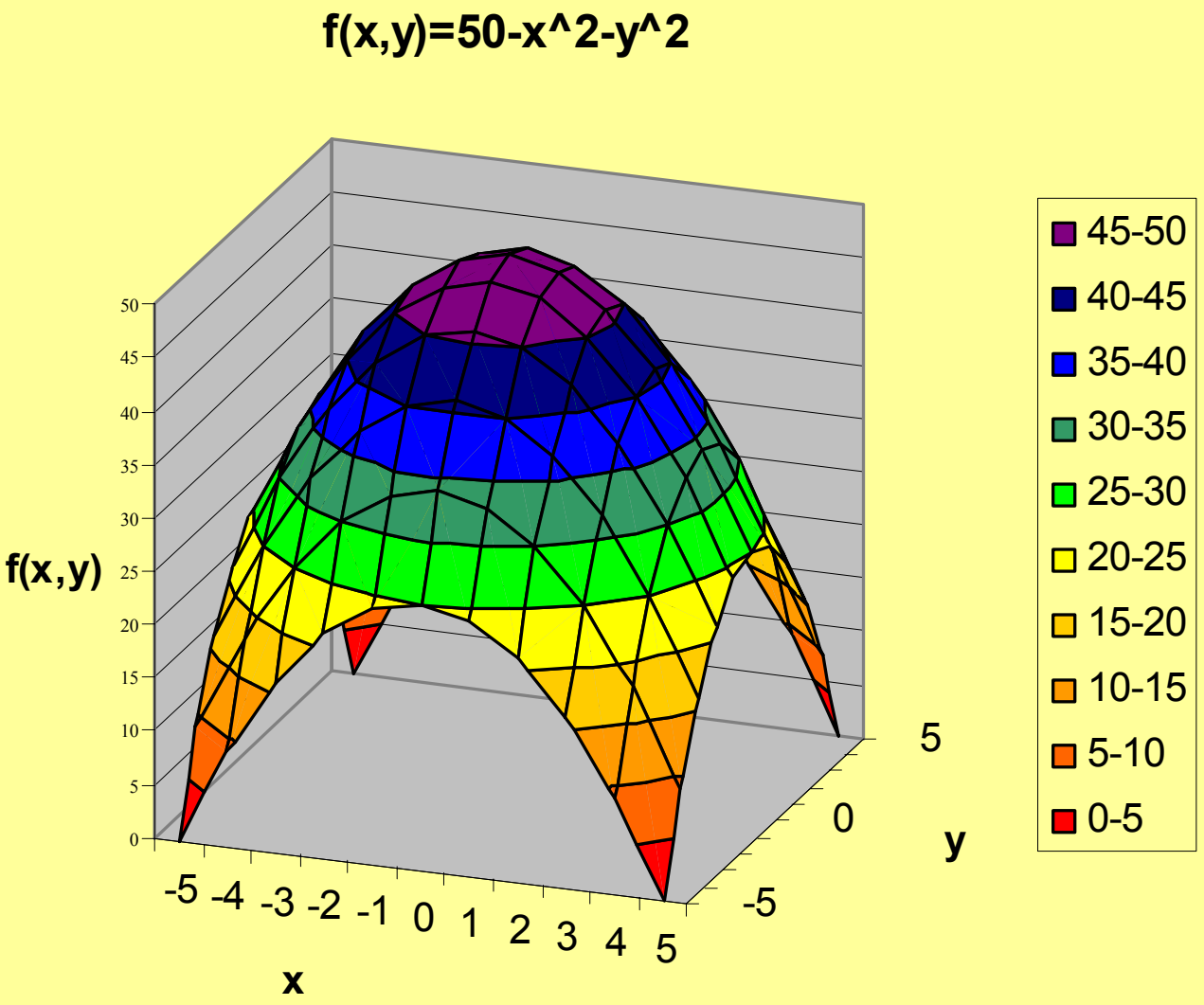
Visualization—Table

	-5	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**
-5	0	9	16	21	24	25	24	21	16	9	0
-4	9	18	25	30	33	34	33	30	25	18	9
-3	16	25	32	37	40	41	40	37	32	25	16
-2	21	30	37	42	45	46	45	42	37	30	21
-1	24	33	40	45	48	49	48	45	40	33	24
0	25	34	41	46	49	50	49	46	41	34	25
1	24	33	40	45	48	49	48	45	40	33	24
2	21	30	37	42	45	46	45	42	37	30	21
3	16	25	32	37	40	41	40	37	32	25	16
4	9	18	25	30	33	34	33	30	25	18	9
5	0	9	16	21	24	25	24	21	16	9	0

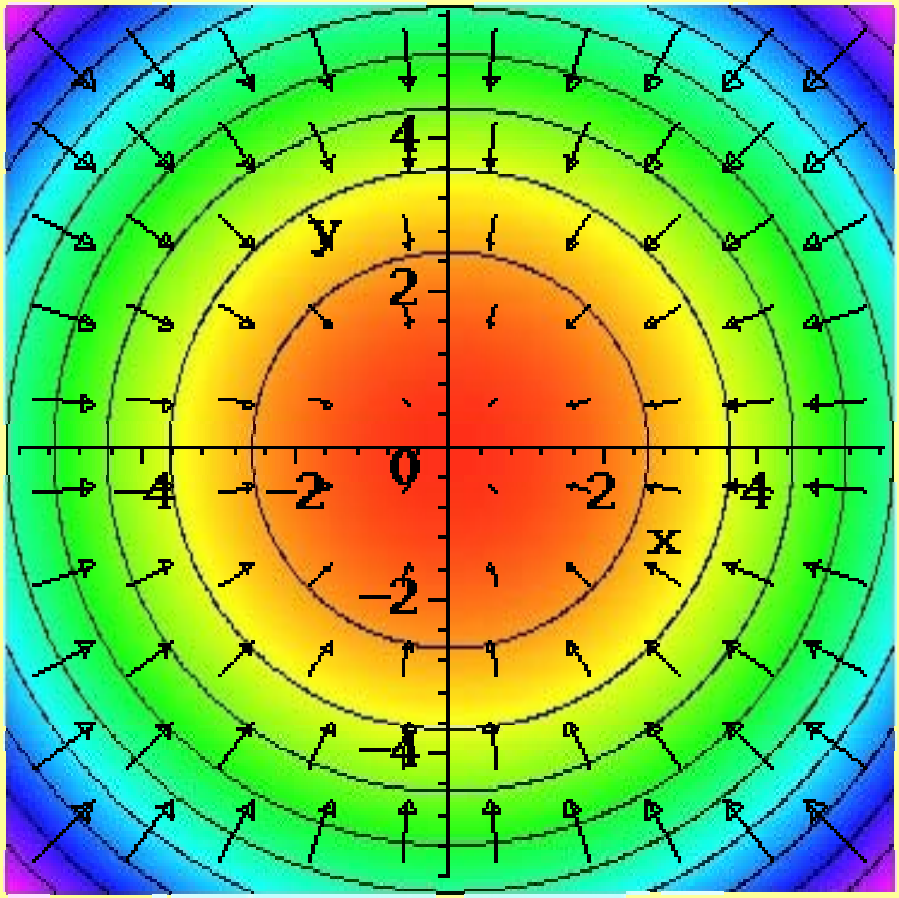
Visualization—Level Curves



Visualization—Graph



Visualization—Gradient

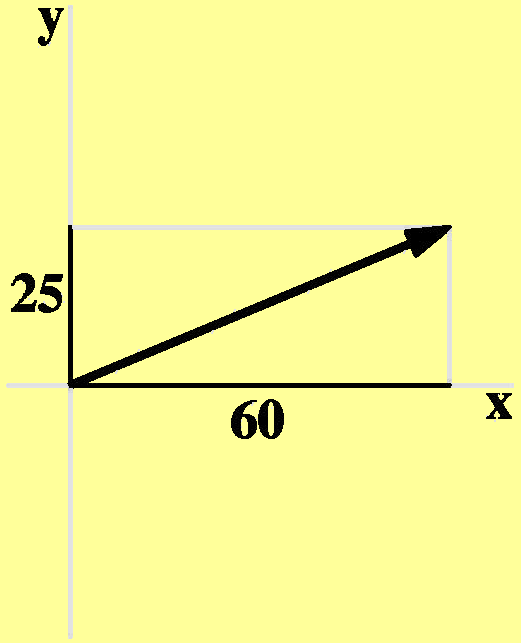


What Are Vectors?

Mathematics:

- Triples of numbers

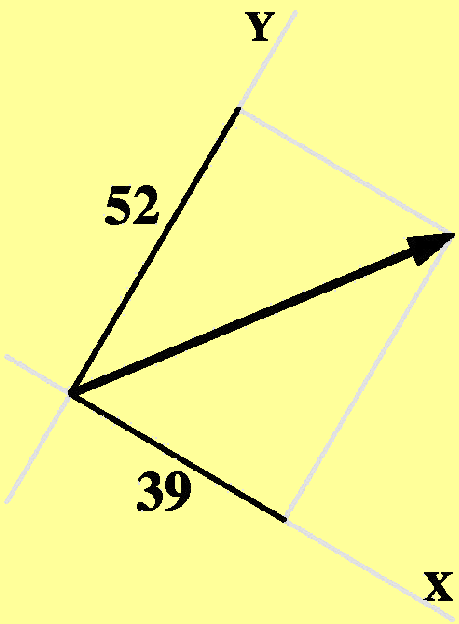
$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$



Physics:

- Arrows in space

$$\vec{v} = v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}$$



Do You Do This?

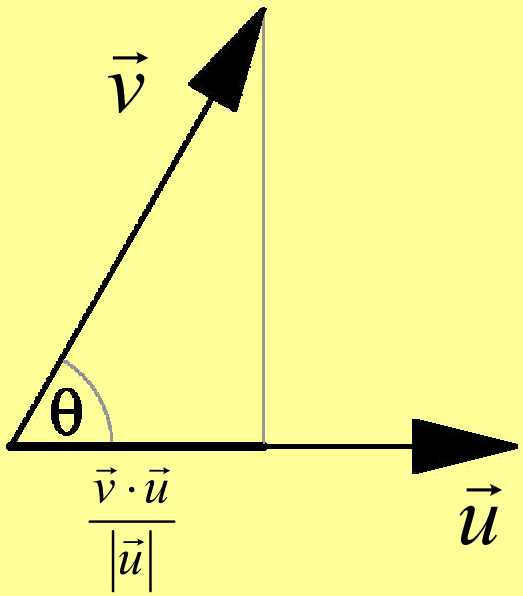
$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$\left|\vec{u}\right| = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$$

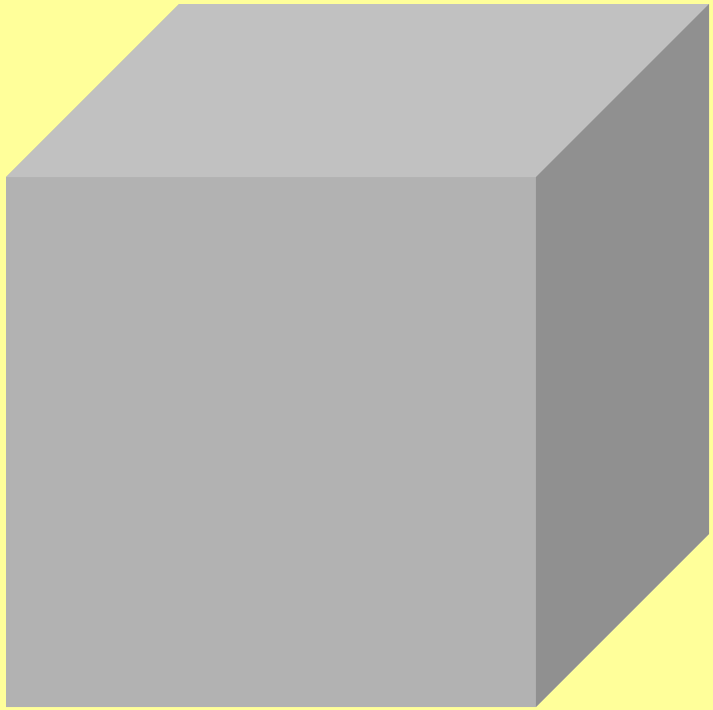
Or This?

$$\vec{u} \cdot \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

$$\left|\vec{u}\right| = \sqrt{\vec{u} \cdot \vec{u}}$$



Group Activity



Find the angle between a diagonal of a cube and an edge

Group Activity

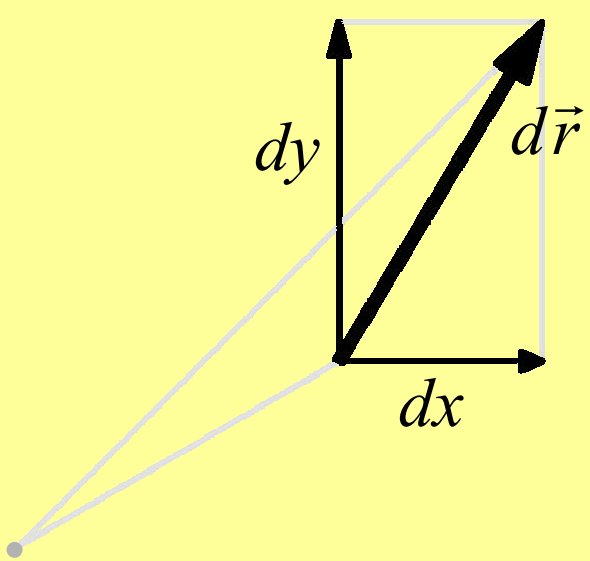
- Emphasizes that vectors are arrows
- Combines geometry and algebra
- Uses multiple representations

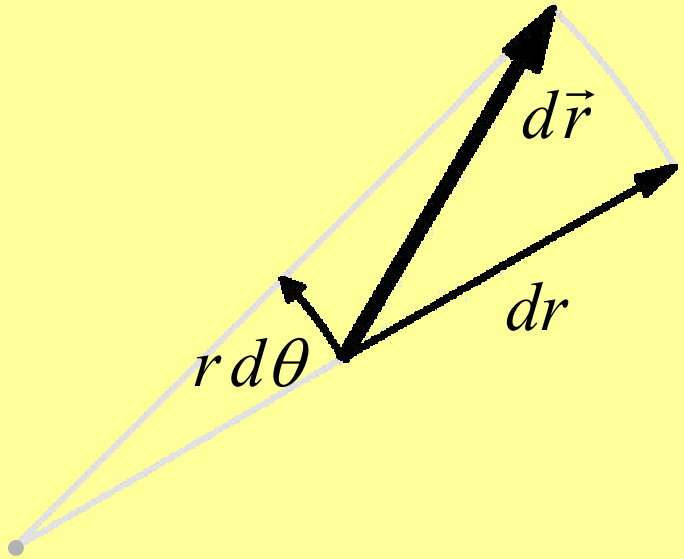
geometry: $\vec{u} \cdot \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right| \cos\theta$

algebra: $\vec{u} \cdot \vec{v} = \left(u_1 \hat{\imath} + u_2 \hat{\jmath} + u_3 \hat{k}\right) \cdot \left(v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{k}\right)$

memory: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Integrals

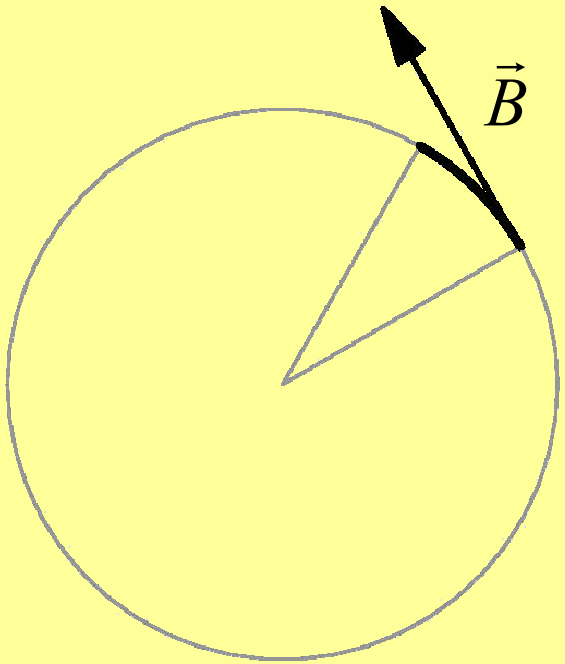




$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

Symmetry



$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r}\hat{\theta}$$

$$d\vec{r} = r\,d\theta\,\hat{\theta}$$

$$\int \vec{B}\cdot d\vec{r} = \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\theta = \mu_0 I$$

Ignoring Symmetry

$$\vec{B} = \frac{\mu_0 I}{2\pi} \frac{-y\,\hat{\imath} + x\,\hat{\jmath}}{x^2 + y^2}$$

$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned} \quad \Rightarrow \quad \begin{aligned} dx &= -r\sin\theta\, d\theta \\ dy &= r\cos\theta\, d\theta \end{aligned}$$

$$\int \vec{B} \cdot d\vec{r} = \int_0^{2\pi} \frac{\mu_0 I}{2\pi} d\theta = \mu_0 I$$

The Bridging Project

- Differentials (Use what you know!)
- Exploiting multiple representations
- Symmetry (adapted bases, coordinates)
- Geometry (vectors, div, grad, curl)
- Computer visualization (Maple, Excel)
- Active engagement (groups, flashcards)