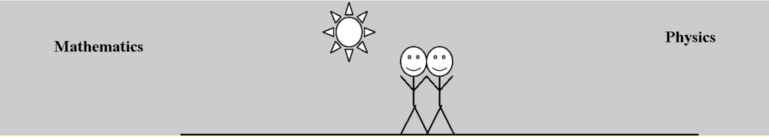
Bridging the Gap between Mathematics and the Physical Sciences

Tevian Dray & Corinne A. Manogue

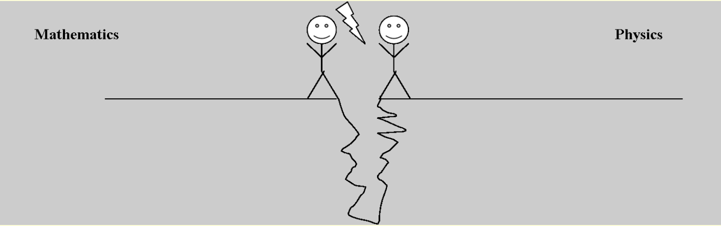
Oregon State University

Mathematics vs. Physics

Mathematics vs. Physics



Mathematics vs. Physics



What are Functions?

Suppose the temperature on a rectangular slab of metal is given by

$$T(x,y) = k(x^2 + y^2)$$

where k is a constant. What is $T(r,\theta)$?

What are Functions?

Suppose the temperature on a rectangular slab of metal is given by

$$T(x, y) = k(x^2 + y^2)$$

where k is a constant. What is $T(r, \theta)$?

A: $T(r, \theta) = kr^2$

What are Functions?

Suppose the temperature on a rectangular slab of metal is given by

$$T(x, y) = k(x^2 + y^2)$$

where k is a constant. What is $T(r, \theta)$?

A: $T(r, \theta) = kr^2$

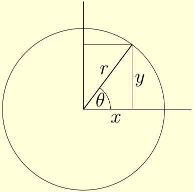
B: $T(r, \theta) = k(r^2 + \theta^2)$

What are Functions?

Suppose the temperature on a rectangular slab of metal is given by

$$T(x, y) = k(x^2 + y^2)$$

where k is a constant. What is $T(r, \theta)$?



A: $T(r, \theta) = kr^2$

B: $T(r, \theta) = k(r^2 + \theta^2)$

What are Functions?

MATH

$$
\begin{aligned}
T &= f(x,y) = k(x^2+y^2) \\
T &= g(r,\theta) = kr^2
\end{aligned}
$$

What are Functions?

MATH

$$
\begin{aligned}
T &= f(x,y) = k(x^2+y^2) \\
T &= g(r,\theta) = kr^2
\end{aligned}
$$

PHYSICS

$$
\begin{aligned}
T &= T(x,y) = k(x^2+y^2) \\
T &= T(r,\theta) = kr^2
\end{aligned}
$$

What are Functions?

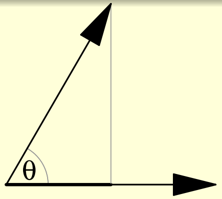
MATH

$$\begin{aligned} T &= f(x,y) = k(x^2+y^2) \\ T &= g(r,\theta) = kr^2 \end{aligned}$$

PHYSICS

$$\begin{aligned} T &= T(x,y) = k(x^2+y^2) \\ T &= T(r,\theta) = kr^2 \end{aligned}$$

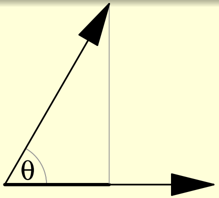
Two disciplines separated by a common language...

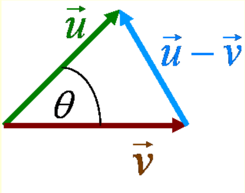


Projection:

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}| \cos\theta$$

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = u_x v_x + u_y v_y$$





Projection:

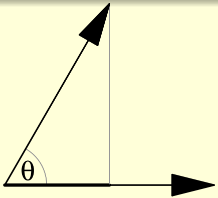
$$\vec{\mathbf{u}}\cdot\vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}|\cos\theta$$

$$\vec{\mathbf{u}}\cdot\vec{\mathbf{v}} = u_x v_x + u_y v_y$$

Law of Cosines:

$$(\vec{\mathbf{u}}-\vec{\mathbf{v}})\cdot(\vec{\mathbf{u}}-\vec{\mathbf{v}}) = \vec{\mathbf{u}}\cdot\vec{\mathbf{u}} + \vec{\mathbf{v}}\cdot\vec{\mathbf{v}} - 2\vec{\mathbf{u}}\cdot\vec{\mathbf{v}}$$

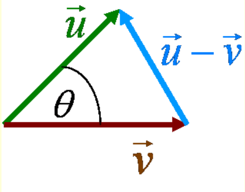
$$|\vec{\mathbf{u}}-\vec{\mathbf{v}}|^2 = |\vec{\mathbf{u}}|^2 + |\vec{\mathbf{v}}|^2 - 2|\vec{\mathbf{u}}||\vec{\mathbf{v}}|\cos\theta$$



Projection:

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}| \cos\theta$$

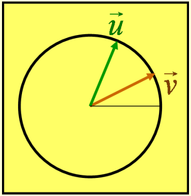
$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = u_x v_x + u_y v_y$$



Law of Cosines:

$$(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} - \vec{\mathbf{v}}) = \vec{\mathbf{u}} \cdot \vec{\mathbf{u}} + \vec{\mathbf{v}} \cdot \vec{\mathbf{v}} - 2\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$$

$$|\vec{\mathbf{u}} - \vec{\mathbf{v}}|^2 = |\vec{\mathbf{u}}|^2 + |\vec{\mathbf{v}}|^2 - 2|\vec{\mathbf{u}}||\vec{\mathbf{v}}| \cos\theta$$



Addition Formulas:

$$\vec{\mathbf{u}} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$$
$$\vec{\mathbf{v}} = \cos\beta\,\hat{\imath} + \sin\beta\,\hat{\jmath}$$
$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \cos(\alpha - \beta)$$

Find the angle between the diagonal of a cube and one of its edges.

Find the angle between the diagonal of a cube and one of its edges.

Algebra:

$$\begin{aligned}\vec{\mathbf{u}} &= \hat{\imath} + \hat{\jmath} + \hat{k} \\ \vec{\mathbf{v}} &= \hat{\imath}\end{aligned}$$

$$\Longrightarrow \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 1$$

Geometry:

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}| \cos\theta = \sqrt{3}\cos\theta$$

Find the angle between the diagonal of a cube and one of its edges.

Algebra:

$$\begin{aligned} \vec{\mathbf{u}} &= \hat{\imath} + \hat{\jmath} + \hat{k} \\ \vec{\mathbf{v}} &= \hat{\imath} \end{aligned}$$

$$\Longrightarrow \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 1$$

Geometry:

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}| \cos\theta = \sqrt{3} \cos\theta$$

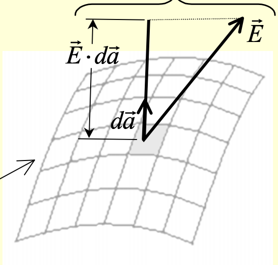
Need both!

Flux is the total amount of electric field through a given area.

Perpendicular Component

Normal Vector

$$\Phi = \int \vec{E} \cdot d\vec{a}$$



$\sum$ over all rectangles

Kerry Browne (Ph.D. 2002)

The Vector Calculus Bridge Project

http://www.math.oregonstate.edu/bridge

The Vector Calculus Bridge Project

- **Differentials** *(Use what you know!)*
- **Multiple representations**
- **Symmetry** *(adapted bases, coordinates)*
- **Geometry** *(vectors, div, grad, curl)*

`http://www.math.oregonstate.edu/bridge`

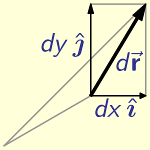
The Vector Calculus Bridge Project

- **Differentials** *(Use what you know!)*
- **Multiple representations**
- **Symmetry** *(adapted bases, coordinates)*
- **Geometry** *(vectors, div, grad, curl)*

- Small group activities
- Instructor's guide (in preparation)

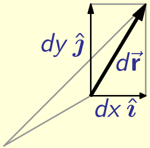
`http://www.math.oregonstate.edu/bridge`

Vector Differentials

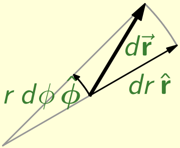


$$d\vec{\mathbf{r}} = dx\,\hat{\boldsymbol{\imath}} + dy\,\hat{\boldsymbol{\jmath}}$$

Vector Differentials

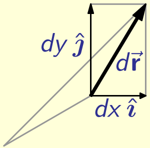


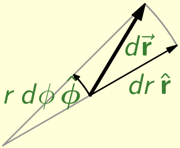
$$d\vec{\mathbf{r}} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$



$$d\vec{\mathbf{r}} = dr\,\hat{\mathbf{r}} + r\,d\phi\,\hat{\boldsymbol{\phi}}$$

Vector Differentials





$$d\vec{\mathbf{r}} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$d\vec{\mathbf{r}} = dr\,\hat{\mathbf{r}} + r\,d\phi\,\hat{\phi}$$

$$\boxed{\begin{aligned} ds &= |d\vec{\mathbf{r}}| \\ d\vec{\mathbf{A}} &= d\vec{\mathbf{r}}_1 \times d\vec{\mathbf{r}}_2 \\ dA &= |d\vec{\mathbf{r}}_1 \times d\vec{\mathbf{r}}_2| \\ dV &= (d\vec{\mathbf{r}}_1 \times d\vec{\mathbf{r}}_2) \cdot d\vec{\mathbf{r}}_3 \end{aligned}}$$

Gradient

Gradient

Master Formula:

$$df = \vec{\nabla} f \cdot d\vec{\mathbf{r}}$$

Gradient

Master Formula:

$$df = \vec{\nabla} f \cdot d\vec{\mathbf{r}}$$

$$f = \text{const} \implies df = 0 \implies \vec{\nabla} f \perp d\vec{\mathbf{r}}$$

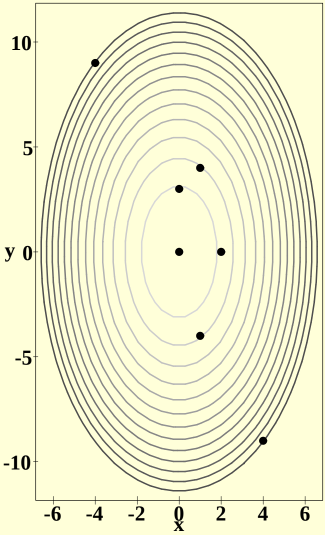
$$\frac{df}{ds} = \vec{\nabla} f \cdot \frac{d\vec{\mathbf{r}}}{|d\vec{\mathbf{r}}|}$$

The Hill

Suppose you are standing on a hill. You have a topographic map, which uses rectangular coordinates (x, y) measured in miles. Your global positioning system says your present location is at one of the points shown. Your guidebook tells you that the height h of the hill in feet above sea level is given by

$$h = a - bx^2 - cy^2$$

where $a = 5000$ ft, $b = 30\,\frac{\text{ft}}{\text{mi}^2}$, and $c = 10\,\frac{\text{ft}}{\text{mi}^2}$.



Small Groups

- Forming small groups
- Whiteboards
- Controlling the pen

Classroom Norms

- Open-ended questions
- Level of difficulty
- When can I leave?
- Wrapup
- Control

Other Types of Active Engagement

- Small whiteboards
- Kinesthetic activities
- Computer visualization

Course Structure

- Integrating activities
- TAs

SUMMARY

SUMMARY

I took this class a year ago, and I still remember all of it...